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RECURSIVE NAVIGATION ALGORITHM FOR ELECTRONIC GRID INTEGRATION

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INTRODUCTION

OVERVIEW

A time division multiple access (TDMA) communications system, which supports the communications among all elements within line-of-sight of one another, offers, with its inherent ranging capability, the potential for increased position accuracy for each participant. In the Electronic Grid Integration Program, the combination of Loran inputs with the time-of-arrival (TOA) inputs from the TDMA communications network was examined. This paper defines the software design relating to the position location and synchronization algorithm in use during the final flight tests of that program.

The algorithm is basically similar to that used in the computer simulation described in Reference 1. There are some important exceptions, however. The "live" program maintained only a single clock model. This required that the TDMA system master synchronize to the Loran net, so that passive range measurements on Loran signals could be based on the system clock model. This would limit an operational system to the use of TDMA data plus one "other" kind of data source. In reality, it would be a fairly simple matter to maintain several different clock models so as to remove such a limitation. A sophisticated control logic, which included reference feedback, was quite probably better left out. While it was shown in Reference 1, that in particular situations some advantage was gained, it must be confessed that the implications of feedback control applied to a multi-element system are not fully appreciated.

Omitted from both the simulation and the "live" program, is a position search phase. This would be a definite requirement in an operational system, as it simplifies the start-up process (from an operator point of view) and warns of possible ambiguous position solutions.

The program is a natural and logical outgrowth of the recursive navigation simulation (Reference 2) development which was begun early in 1969. The salient aspects of recursive navigation as applied to the Electronic Grid Integration Program are discussed in the pages immediately following in this section. In Section II, the theoretical considerations of the algorithm are discussed in some detail. Program interrupts, by which means the cycling process is maintained, are introduced in Section III. The routines and subroutines of the position location and synchronization algorithm are presented in Sections IV and V. Finally, an alphabetic parameter listing is provided in Section VI.

FRAME OF REFERENCE

The frame of reference for navigation within the recursive navigation scheme is an ellipsoid of revolution. Each unit, or element of the system, is represented as being at an altitude Z above the point $\emptyset$, λ (latitude, longitude) on the surface of the ellipsoid. The ellipsoid is taken to be the 1966 Smithsonian Standard Earth, usually referred to as SAO 1966 (Reference 3). This ellipsoid is defined as having a semi-major axis (equatorial radius) of $a = 6{,}378{,}165$ meters, and a flattening defined by $1/f = 298.25$. Airborne units are assumed to have a velocity vector, speed V and heading H, oriented in a plane tangent to the surface at the point $\emptyset$, λ. The relation, one nautical mile = exactly 1852 meters, is assumed.

PERIODIC POSITION REPORT

The key to recursive navigation is the periodic position report which each unit transmits once per frame in its assigned slot within the time division multiple access (TDMA) communications network. This report always contains position (no matter how poorly known) and it is always transmitted once each frame (unless in the QUIET mode) subsequent to the receipt of a start command and slot assignment. The report also contains the unit state and uncertainty matrix, which indicate the suitability of the unit as a data source. The former provides a simple yes/no condition, while the latter defines in what manner the data can best be utilized.

During the frame, each unit endeavors to determine as precisely as possible the proper time to transmit the next report, and the position and other information it should contain. Loran inputs and/or position reports from other TDMA units will be utilized in that determination. Other information in the position report will include identification, task, slot number, speed, heading, and altitude.

UNIT TASK DESIGNATION

There are four different unit task designations corresponding to the four different types of units in the TDMA system. They are:

- MASTER - provides the system time and position reference
- REFERENCE - a sited, stationary ground station
- GROUND - an unsited ground station, stationary or quasi-stationary
- AIRBORNE - any unit in motion

The MASTER monitors the system and sends feedback messages causing collective adjustments in timing and position estimates by the GROUND and AIRBORNE system members.

A REFERENCE unit adjusts timing estimates only; its position is assumed known.

A GROUND unit has a zero velocity vector. It adjusts timing and position estimates.

An AIRBORNE unit has a non-zero velocity vector. It adjusts timing, position and velocity estimates.

MODES OF OPERATION

There are three possible modes of operation for any unit except MASTER. These are:

RANGING - utilizes round trip ranging
NORMAL - utilizes passive one-way ranging
QUIET - same as NORMAL except does not transmit

The operating mode is determined under program control which considers a preferred mode based on available data sources as well as the mode selected by manual switch action. There are four possible switch selections. These are RANGING, NORMAL, QUIET and UNSELECTED. With the latter switch setting, the program will operate in the preferred mode.

In the RANGING mode, a unit periodically interrogates others to obtain round trip ranges. In this mode, position location and time synchronization are performed independently. Unless specifically selected, this mode is used only when the number of available data sources cannot support NORMAL mode operation.

In the NORMAL mode, periodic position reports and Loran inputs are used to determine one-way ranges. Adjustments to position and timing estimates are determined simultaneously.

The QUIET mode is similar to the NORMAL mode except that the local unit does not transmit.

Initially, the QUIET mode is forced upon all units except MASTER, regardless of switch selection, to prevent interference. Not until at least one position report has been received from a source claiming synchronization is a change to the NORMAL or RANGING operating mode permitted.

PHASING SCHEME

The recursive process of updating the uncertainty matrices and the state vector modifies previous assumptions of state based on current measurements. This requires an initializing process, so that the recursion can get under way. The initialization is divided into three short phases followed by the final recursive phase. Briefly, the phasing scheme is:

(0) Gross Sync -- to obtain, or reacquire, rough clock synchronization;

(1) Position Search -- to obtain a "reasonably close" position estimate, and to warn the operator of possible ambiguous position solutions;

(2) Start Up -- to provide the initial values for the time and position states and the associated covariance matrices; and

(3) Recursive -- to modify previous assumptions of state based on current measurements, and update the uncertainty matrices and the state vector.

The position search phase was never programmed. In its place, a "reasonably close" starting position must be manually entered during program initialization (reasonably close can be taken to mean within 5 or 10 miles and in the right quadrant, i.e., the appropriate general direction from the data sources). This was satisfactory for the purposes of Project 7200; however, in an operational system, the position search phase is an absolute necessity. Position search consists of iterations over the same set of observations, from several well chosen starting points, to find a convergent solution, or possibly ambiguous convergent solutions.

DATA PRIORITY

A two level data priority scheme is followed which utilizes the MIS indicator. (Originally, the indicator literally meant "Master-In-Sight", but evolved to mean "Master-In-Control".) All units will process position reports only from sources with the MIS indicator set to one. Airborne and ground units, in fact, set MIS to one in any frame that the master position report is received. Reference units, however, always indicate MIS on in their position report. The master then monitors only position reports with MIS on for feedback purposes.

The first level of data priority includes all interacting elements contributing to, and under the influence of, master feedback control. The second level includes non-interacting elements outside of the master unit's influence, which utilize first level data priority sources. This simple priority scheme was quite adequate for the limited scope of the Project 7200 experimental area. However, an operational system should extend the scheme to at least four levels of data priority, as described in References 1 and 2.

UNIT STATE (OR DATA STATUS)

There are four possible states that a unit may be in (and thus his data, if he transmits). The states are determined automatically under program control. They are:

- UU - time and position Unknown
- UP - time Unknown, Position known
- SU - time Synchronized, position Unknown
- SP - time Synchronized, Position known

AIRBORNE and GROUND units will start in state UU. Normally they will progress directly to state SP, though, on occasion, it may be desirable to achieve status UP or SU in the RANGING mode.

REFERENCE sites will start in state UP. If the master unit is being received, they will process only master data. Failing this, if one or more references are being received, a reference will process only other reference data. If no master or reference data are available, then they will process ground and airborne data.

STATE CHECK

Initially, and until state SP (Synchronized, Position known) is attained <u>and</u> the unit is operating in the NORMAL mode, it is attempted to upgrade the unit state changing the mode of operation, if desirable and permitted. Then, while in state SP, mode NORMAL, it is attempted to maintain the status quo. If analysis of the data sources indicates that state cannot be upgraded (or maintained in SP) in the present mode, but can be upgraded or maintained in another, then the mode of operation will be changed, unless this conflicts with the mode switch selection. If data become sparse, the RANGING mode may be attempted in order to achieve or maintain state SP.

Two counters are maintained for the purpose of upgrading and downgrading status, both of which represent a length of time spent in some condition. Associated with each counter are transition values, which, when attained, will cause a change in state or mode. The counters are stepped, or reset, in conjunction with time and position solution indicators, TN and PN. These are set to +1 or 0 during end of frame processing to indicate if a solution was obtained. The indications are:

TN $\begin{cases} 1 & \text{satisfactory time solution} \\ 0 & \text{insufficient data or poor geometry} \end{cases}$

PN $\begin{cases} 1 & \text{satisfactory position solution} \\ 0 & \text{insufficient data or poor geometry} \end{cases}$

TN and MIS are set to zero, should the mean time discrepancy exceed program limits. In states SU and SP, a normal position solution may be obtained with only two data sources.

CT Counter

A unit in time state S, counts (negatively) consecutive frames with no time solution. The counter is reset to -1 in any frame with a satisfactory solution (TN = 1). Insufficient data (TN = 0) causes the counter to be stepped downward, and if it reaches -CTU, the time state is changed to U, and the phase is changed to gross sync.

In time state U, insufficient data continues to step the counter downward, and should it reach -CTQ, the QUIET mode is forced on the unit. A single satisfactory solution (TN = 1) resets the counter to +1 and starts the resynchronizing process, during which satisfactory solutions step the counter up and insufficient data steps the counter down (skipping zero). Upon reaching +CTS, the time state is changed to S.

Initially, the unit's mode is QUIET (except MASTER) and CT is zero. However, a receipt of a single synchronized datum is sufficient to initiate the synchronization process in either the NORMAL or RANGING mode.

CP Counter

In position state U, the counter is stepped up in any frame that position data is processed satisfactorily (PN = 1), and stepped down (not below zero) when insufficient data occurs (PN = 0). If the counter reaches CPK, the state is changed to P.

Airborne units in position state P use this counter to count consecutive frames with no satisfactory solution (PN = 0). The counter is reset to zero in any frame that position data are processed satisfactorily (PN = 1). If the counter reaches +CPU, the state is changed to U, and the position phase is returned to start up.

Ground units, having achieved state P, remain there.

DATA SOURCE TABLE

Each unit maintains a data source listing and updates it each frame. Whenever a position report is received, the sender identification, arrival time, slot number, position and other pertinent information is entered into the table. Messages other than position reports are not entered. Units will use the table to determine possible mode, or state change, and to determine sources worthy of interrogation in the ranging mode.

In the Project 7200 experimental area, the table was indexed by prestored ID, since only a small number (10) of potential sources existed. In an operational system it would be pointless and costly to record data on every position report received. However, it will be useful to maintain a time ordered list of from eight to sixteen potential sources to be interrogated in the ranging mode. Quite likely, the data source table could be a part of, or an extension of the ranging interrogation table. (See the Ranging Interrogation Set-Up Subroutine in Section V.)

SECTION II

THEORETICAL CONSIDERATIONS

GENERAL

Each unit maintains an assumed state vector and a covariance matrix of the uncertainty of the estimates of system time and position. Taking x and y in the direction of positive west longitude and north latitude, these can be represented as:

$$S = \begin{bmatrix} t \\ \dot{t} \\ x \\ \dot{x} \\ y \\ \dot{y} \\ z \end{bmatrix} \qquad C = \begin{bmatrix} \sigma_{tt}^2 & \sigma_{tx}^2 & \sigma_{ty}^2 \\ \sigma_{tx}^2 & \sigma_{xx}^2 & \sigma_{xy}^2 \\ \sigma_{ty}^2 & \sigma_{xy}^2 & \sigma_{yy}^2 \end{bmatrix} \tag{1}$$

All but the time variables are transmitted each frame. The variables time and time-rate, t and $\dot{t}$, in fact determine when the transmission occurs and serves as a basis for making time-of-arrival measurements on other position reports. If any unit is to utilize Loran inputs, this then requires that the TDMA system master synchronize to the Loran net. This is no problem; however, alternatively, separate clock models could be maintained -- one for system time and one for Loran time. The velocity variables apply only to aircraft and are in terms of speed and heading, V and H. The state vector and the uncertainty matrix are updated at the end of each frame, based on the data received during the frame. Altitude is not filtered -- it represents the current air data input or manual entry, as applicable. For convenience of notation, the covariance matrix will be written as:

$$C = \begin{bmatrix} stt & stx & sty \\ stx & sxx & sxy \\ sty & sxy & syy \end{bmatrix} \tag{2}$$

To support the process, each unit maintains a clock covariance matrix correlating the uncertainty in system time and time-rate.

$$C_t = \begin{bmatrix} \sigma_{tt}^2 & \sigma_{t\dot{t}}^2 \\ \sigma_{\dot{t}t}^2 & \sigma_{\dot{t}\dot{t}}^2 \end{bmatrix} \tag{3}$$

This is updated recursively at the end of each frame based on data received during the frame. Also, airborne units maintain the position covariance matrices

$$C_x = \begin{bmatrix} \sigma_{xx}^2 & \sigma_{x\dot{x}}^2 \\ \sigma_{\dot{x}x}^2 & \sigma_{\dot{x}\dot{x}}^2 \end{bmatrix} \qquad C_y = \begin{bmatrix} \sigma_{yy}^2 & \sigma_{y\dot{y}}^2 \\ \sigma_{\dot{y}y}^2 & \sigma_{\dot{y}\dot{y}}^2 \end{bmatrix} \tag{4}$$

relating position and velocity in the x and y (west and north) directions. Ground units, having no velocity vector, simply update σ_{xx}^2 and σ_{yy}^2, which are the same as the corresponding elements sxx and syy of the C matrix. The three matrices, C_t, C_x, and C_y, will, for convenience, be written

$$\begin{bmatrix} at & bt \\ bt & ct \end{bmatrix} \qquad \begin{bmatrix} ax & bx \\ bx & cx \end{bmatrix} \qquad \begin{bmatrix} ay & by \\ by & cy \end{bmatrix} \tag{5}$$

These matrices are not transmitted, but are used in the process of computing the six different elements of the uncertainty matrix that is transmitted.

MEASUREMENT AND ERROR ESTIMATION

The position report, transmitted at time t by each unit, contains (at least) the three position elements Ø, λ and Z, latitude, longitude and altitude, and the six covariance elements stt, stx, sty, sxx, sxy and syy. Collectively, these position reports become potential observations for every other unit in the system. Each unit references his observations to a plane tangent to the spheroid at his assumed position, so that differences of longitude and latitude become differences in x and y. Differential corrections, determined in the plane in terms of x and y are then converted to adjustments in longitude and latitude. Details of these conversions are contained in Parameters and Geometry subroutines and will be omitted here.

In Normal mode operation, an observation is of the form

$$\theta_i = (t_i - t_o(i))\nu p - \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} \tag{6}$$

where νp is the electromagnetic velocity of propagation. More simply, the observation is just

$$\theta_i = Rt_i - Rc_i \tag{7}$$

that is, time measured one way range less the apparent position difference, or computed range.

Defining a unit's transmission time error as

$$et = T_{(proper)} - t_{(assumed)} \tag{8}$$

the range error due to error in time will be

$$er = et \cdot \nu p \tag{9}$$

and, if position error is defined as

$$\begin{bmatrix} ex \\ ey \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}_{(assumed)} - \begin{bmatrix} X \\ Y \end{bmatrix}_{(true)} \tag{10}$$

then the observation is related to the state error by

$$\theta_i = \begin{bmatrix} 1 & cx_i & cy_i \end{bmatrix} \cdot \begin{bmatrix} er \\ ex \\ ey \end{bmatrix} \tag{11}$$

where

$$\begin{aligned} cx_i &= (x_i - x)/Rc_i \\ cy_i &= (y_i - y)/Rc_i \end{aligned} \tag{12}$$

A term, WT_i, sensitive to the type of source is determined by the receiving element. The TDMA system master, reference units, and Loran inputs are given the value $WT_i = 1$; ground units, $WT_i = 2$; and airborne units, $WT_i = 4$.

(While the above simple assignment scheme may be satisfactory for the small scale test area of Project 7200, in an operational system it will be necessary to set WT_i proportional to the number of sources being utilized. For example:

$$\begin{aligned} &\text{If the data source is a ground unit, } WT_i = 1 + N/6 \\ &\text{If the data source is airborne, then } WT_i = 1 + N/3 \end{aligned} \tag{12a}$$

This prevents a large number of airborne and ground units from swamping the master and reference data. Loran data could, and probably should be assigned values of WT_i that are proportional to the apparent time-of-arrival uncertainty of each source. The signal-to-noise ratio might well be used for this purpose.)

Each position report to be utilized (some may be screened out) is given a weight, according to

$$\zeta_i = WT_i + stt_i + 2cx_i stx_i + 2cy_i sty_i + cx_i^2 sxx_i + 2cx_i cy_i sxy_i + cy_i^2 syy_i \tag{13}$$

which is inverted to form the weighting factor

$$w_i = 1/\zeta_i \tag{14}$$

The inverse weights form a diagonal weighting matrix

$$W = \begin{bmatrix} w_1 & & & & & 0 \\ & w_2 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & 0 & & & \cdot & \\ & & & & & w_n \end{bmatrix} \tag{15}$$

while the observations and direction cosines form the matrices

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \qquad J = \begin{bmatrix} 1 & cx_1 & cy_1 \\ 1 & cx_2 & cy_2 \\ \vdots & \vdots & \vdots \\ 1 & cx_n & cy_n \end{bmatrix} \tag{16}$$

The minimum variance solution for the estimated error vector is given by

$$\hat{e} = (J^T W J)^{-1} \cdot J^T W \theta \tag{17}$$

and the variances of the error estimates are provided by the diagonal elements of $(J^T WJ)^{-1}$ (Reference 4).

As observations arrive, they are collected in running sums.

$$
\begin{aligned}
N &= N + w_i \\
NS &= NS + 1 \\
cx &= cx + w_i \cdot cx_i \\
cy &= cy + w_i \cdot cy_i \\
cxy &= cxy + w_i \cdot cx_i \cdot cy_i \\
CX2 &= CX2 + w_i \cdot {cx_i}^2 \\
CY2 &= CY2 + w_i \cdot {cy_i}^2 \\
d &= d + w_i \cdot \theta_i \\
dx &= dx + w_i \cdot cx_i \cdot \theta_i \\
cy &= cy + w_i \cdot cy_i \cdot \theta_i
\end{aligned}
\tag{18}
$$

At the end of a frame then, these sums represent the six elements of (J^TWJ) and the three elements of $J^TW\theta$. Representing the estimated error vector $\hat{e}$ as the differential corrections DR, DX and DY in range and position, equation (17) now appears as

$$
\begin{bmatrix} DR \\ DX \\ DY \end{bmatrix} = \begin{bmatrix} N & cx & cy \\ cx & CX2 & cxy \\ cy & cxy & CY2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} d \\ dx \\ dy \end{bmatrix} \tag{19}
$$

At ground and airborne units, the single fix, independent estimate of range error, DR, is found as usual by

$$
\begin{aligned}
E1 &= CX2 \cdot CY2 - (cxy)^2 \\
E2 &= cy \cdot cxy - cx \cdot CY2 \\
E3 &= cx \cdot cxy - cy \cdot CX2 \\
\nabla &= N \cdot E1 + cx \cdot E2 + cy \cdot E3
\end{aligned}
\tag{20}
$$

then

$$DR = (E1 \cdot d + E2 \cdot dx + E3 \cdot dy)/\nabla \tag{21}$$

which, upon division by νp, propagation velocity, becomes the estimated system clock error in units of time. This is usually referred to as the (3 x 3) solution for time. Its variance is given by

$$\sigma_{3t}^2 = E1/\nabla \tag{22}$$

Position estimates DX and DY may be obtained from the (3 x 3), which continues after computing DR,

$$\begin{aligned} X2 &= N \cdot CY2 - (cy)^2 \\ X3 &= cx \cdot cy - N\ cxy \\ Y3 &= N \cdot CX2 - (cx)^2 \end{aligned} \tag{23}$$

then

$$\begin{aligned} DX &= (E2 \cdot d + X2 \cdot dx + X3 \cdot dy)/\nabla \\ DY &= (E3 \cdot d + X3 \cdot dx + Y3 \cdot dy)/\nabla \end{aligned} \tag{24}$$

and the associated variances are

$$\begin{aligned} \sigma_{3x}^2 &= X2/\nabla \\ \sigma_{3y}^2 &= Y3/\nabla \\ \sigma_{3xy}^2 &= +X3/\nabla \end{aligned} \tag{25}$$

These (3 x 3) position estimates will be obtained in the Normal mode, during the position search and start-up phases at air and ground units and during the recursive phase at ground units.

Airborne units in Normal mode, obtain position estimates from the time dependent (2 x 2) solution, while in the recursive phase. That is

$$\begin{aligned} DX &= (dx \cdot CY2 - dy \cdot cxy)/E1 \\ DY &= (dy \cdot CX2 - dx \cdot cxy)/E1 \end{aligned} \tag{26}$$

the associated variances being

$$\begin{aligned} \sigma_{2x}^2 &= CY2/E1 \\ \sigma_{2y}^2 &= CX2/E1 \\ \sigma_{2xy}^2 &= -cxy/E1 \end{aligned} \tag{27}$$

Reference units obtain the time error simply by

$$DR = d/(N \cdot \nu p) \tag{28}$$

and its variance is

$$\sigma_{1t}^2 = 1/N \tag{29}$$

The Master unit, which monitors all position reports with the MIS indicator set to one, also uses equations (26) and (28) to compute the apparent system error in position and time. These error estimates are then smoothed, and possibly limited, to produce the feedback items TF, XF and YF, which are transmitted with the master position report.

In the ranging mode of operation a time observation is of the form

$$Dt_i = [(t_i - t_o(i)) - (t_j - t_o(j))]/2 \tag{30}$$

where t_j refers to the time measurement made by the unit interrogated and t_i refers to the measurement made by the local unit on the ranging reply. If either unit is in motion, and if speed and heading information, V and H, is included in the position report, an adjustment factor is added to Dt_i which is of the form

$$t' = \frac{t_o(i) - t_o(j)}{2 \cdot \nu p} \; [(Vx + V_j \sin H_j)cx_i + (Vy - V_j \cos H_j)cy_i] \tag{31}$$

The time observation from the i^{th} source is weighted according to

$$\zeta_i = WT_i + stt_i + 2cx_i stx_i + 2cy_i sty_i \tag{32}$$

which is inverted to form the weighting factor

$$w_i = 1/\zeta_i \tag{14}$$

As the time observations arrive they are collected

$$\begin{aligned} Nt &= Nt + w_i \\ dt &= dt + w_i \cdot Dt_i \end{aligned} \tag{33}$$

Then, at the end of the frame, the estimated clock error, in units of time is

$$DR = dt/Nt \tag{34}$$

and the solution variance is

$$\sigma_{Rt}^2 = 1/2Nt \tag{35}$$

The position observation, in the ranging mode is of the form

$$Dp_i = \tfrac{1}{2}[t_i - t_o(i) + t_j - t_o(j)]\nu p - \sqrt{(x_i-x)^2 + (y_i-y)^2 + (z_i-z)^2} \tag{36}$$

and, in ranging the position observation is weighted by

$$\zeta_i = WT_i + cx_i^2 sxx_i + 2cx_i cy_i sxy_i + cy_i^2 syy_i \tag{37}$$

which is inverted to form the weighting factor

$$w_i = 1/\zeta_i \tag{14}$$

The observations are collected in the same running sums as with the normal mode (equation set (18)). The position solution is found as in normal from the (2 x 2), equation (26), and the variances, similarly, equation (27).

INITIALIZING THE RECURSIVE PROCESS

Since the Kalman filter requires prior information, it is necessary to go through a start-up process to get initial values. In this application, the problem is treated as three independent two-variable processes. One for time, t and $\dot{t}$, and two for position x and $\dot{x}$, and y and $\dot{y}$. Only with airborne units are all three processes fully applicable.

The start-up process can be stated as follows, for the time variables: assume that an initial clock state

$$S(o)_t = \begin{vmatrix} t \\ \dot{t} \end{vmatrix}_o \tag{38}$$

has been established. Each estimate of time error, DR_i, obtained at each end of frame while in the start-up phase, is related to the initial error in assumed clock state through the J_o transition matrix

$$\begin{bmatrix} DR_1 \\ DR_2 \\ \vdots \\ DR_N \end{bmatrix} = \begin{bmatrix} 1 & \tau_1 \\ 1 & \tau_2 \\ \vdots & \vdots \\ 1 & \tau_N \end{bmatrix}_o \cdot \begin{bmatrix} et \\ \\ \dot{et} \end{bmatrix}_o \tag{39}$$

that is

$$DR = J_o \cdot e_o \tag{40}$$

where τ_i is the interval since $S(o)_t$ was established. Associated with each DR_i is the solution variance, $\sigma^2_{t_i}$, which is used to weight the error estimate in a manner identical to the weights given observations. That is,

$$w_i = 1/\sigma^2_{t_i} \tag{41}$$

which form the diagonal weighting matrix

$$W = \begin{bmatrix} w_1 & & & \mathit{0} \\ & w_2 & & \\ & & \ddots & \\ \mathit{0} & & & w_N \end{bmatrix} \tag{42}$$

Now, DR_N is the most recent observation and τ_N is the time since the initial state, $S(o)_t$, was established. DR_N is in fact the last error estimate that will be made in the start-up phase. An adjustment will be made to the initial state which will then be extrapolated an interval τ_N forward to provide the initial state $S(n/n)_t$ for the recursive phase. That state is

$$S(n/n)_t = \begin{bmatrix} 1 & \tau_N \\ 0 & 1 \end{bmatrix} \cdot \left[S(o)_t + (J_o^T W J_o)^{-1} \cdot J_o^T WDR \right] \tag{43}$$

and the initial covariance is

$$C(n/n)_t = \begin{bmatrix} at & bt \\ bt & ct \end{bmatrix} \tag{44}$$

where

$$\begin{bmatrix} at & -bt \\ -bt & ct \end{bmatrix} = (J_o{}^T W J_o)^{-1} \qquad (45)$$

Data is computed and saved in the form of running sums until the N observations, spanning the required frames, have been obtained. Then S(n/n) and C(n/n) are computed. The computations are:

Obtain DR_i for ranging or normal mode

$$\text{Set } w_i = 1/\sigma^2_{t_i} \qquad (41)$$

which is ∇/El in normal (non-reference)

or N reference in normal

or 2Nt in ranging mode.

Then

$$\begin{aligned} \delta &= (\Sigma\, w_i)(\Sigma\, w_i \cdot \tau_i{}^2) - (\Sigma\, w_i \cdot \tau_i)^2 \\ at &= (\Sigma\, w_i \cdot \tau_i{}^2)/\delta \\ bt &= (\Sigma\, w_i \cdot \tau_i)/\delta \\ ct &= (\Sigma\, w_i)/\delta \end{aligned} \qquad (46)$$

and

$$\begin{aligned} \Delta t &= at \cdot \Sigma w_i \cdot DR_i - bt \cdot \Sigma w_i \cdot \tau_i \cdot DR_i \\ \Delta d &= ct \cdot \Sigma w_i \cdot \tau_i \cdot DR_i - bt \cdot \Sigma w_i \cdot DR_i \end{aligned} \qquad (47)$$

become the corrections to be applied to $S(o)_t$.

Then

$$S(n/n)_t = \begin{bmatrix} 1 & \tau_N \\ & \\ 0 & 1 \end{bmatrix} \cdot \left[\begin{bmatrix} t_o \\ \\ \dot{t}_o \end{bmatrix} + \begin{bmatrix} \Delta t \\ \\ \Delta d \end{bmatrix} \right] \tag{48}$$

$$C(n/n)_t = \begin{bmatrix} at & bt \\ & \\ bt & ct \end{bmatrix} \tag{49}$$

The start-up process is identical aboard an aircraft for the position coordinates x and y, with appropriate substitutions in equations (38) through (47). The manner of applying the corrections ΔX, $\Delta\dot{X}$, ΔY and $\Delta\dot{Y}$ will follow the pattern of equations (74) through (77), which converts the corrections to changes of latitude and longitude.

For a ground unit, equation set (46) for position reduces to simply

$$\begin{aligned} ax &= 1/\Sigma w_i(x) \\ \Delta x &= (\Sigma w_i \cdot DX_i)/ax \\ ay &= 1/\Sigma w_i(y) \\ \Delta y &= (\Sigma w_i \cdot DY_i)/ay \end{aligned} \tag{50}$$

KALMAN FILTER

The Kalman filter provides the recursive formulas for updating the state vector and the uncertainty matrices. Again, the problem is treated as three independent two-variable processes. One for time, t and $\dot{t}$, and two for position x and $\dot{x}$, and y and $\dot{y}$, where only with the airborne units are all three processes fully applicable.

The recursion formulas can be stated as follows, for the time variables.

$$\text{If at time n, the clock state is } S(n/n) = \begin{bmatrix} t \\ \\ \dot{t} \end{bmatrix} \tag{51}$$

$$\text{and its covariance is } C(n/n) = \begin{bmatrix} \sigma_{tt}^2 & \sigma_{t\dot{t}}^2 \\ \\ \sigma_{t\dot{t}}^2 & \sigma_{\dot{t}\dot{t}}^2 \end{bmatrix} \tag{52}$$

then the predicted clock state at time n+1, based on data available at time n, is

$$S(n+1/n) = A \cdot S(n/n) \quad \text{where } A = \begin{bmatrix} 1 & \tau \\ \\ 0 & 1 \end{bmatrix} \tag{53}$$

and where τ is the time interval from n to n+1.

If DR is the minimum variance time error estimate and σ_{3t}^2 the solution variance, these items are used to bring forward the covariance matrix and state vector.

$$C^{-1}(n+1/n+1) = \frac{1}{k}(A^T)^{-1}C^{-1}(n/n)A^{-1} + \begin{bmatrix} 1/\sigma_{3t}^2 & 0 \\ \\ 0 & 0 \end{bmatrix} \tag{54}$$

$$S(n+1/n+1) = S(n+1/n) + \frac{1}{\sigma_{3t}^2} C(n+1/n+1) \begin{bmatrix} 1 \\ \\ 0 \end{bmatrix} \cdot DR \tag{55}$$

This is equivalent to saying

$$
\begin{aligned}
\dot{t}_{n+1} &= \dot{t}_n + \beta \cdot DR/\tau \\
t_{n+1} &= t_n + \alpha \cdot DR + \tau \cdot \dot{t}_{n+1}
\end{aligned}
\tag{56}
$$

where

$$
\begin{aligned}
\alpha &= \sigma^2_{tt_{n+1}} / \sigma^2_{3t} \\
\beta/\tau &= \sigma^2_{t\dot{t}_{n+1}} / \sigma^2_{3t}
\end{aligned}
\tag{57}
$$

It is however, more convenient to use α and β/α as smoothing constants. Equations (56) and (57) then take the form

$$
\begin{aligned}
\Delta t &= \alpha \cdot DR \\
\dot{t}_{n+1} &= \dot{t}_n + \frac{\beta}{\alpha} \cdot \Delta t/\tau \\
t_{n+1} &= t_n + \Delta t + \tau \cdot \dot{t}_{n+1}
\end{aligned}
\tag{58}
$$

where

$$
\begin{aligned}
\alpha &= \sigma^2_{tt_{n+1}} / \sigma^2_{3t} \\
\frac{\beta}{\alpha\tau} &= \sigma^2_{t\dot{t}_{n+1}} / \sigma^2_{tt_{n+1}}
\end{aligned}
\tag{59}
$$

The factor k determines the rate at which past observations will be discounted. It is always slightly greater than one, but much less than two. In the steady state condition, k is related to the smoothing parameters alpha and beta by

$$
\alpha = \frac{k^2 - 1}{k^2} \quad \text{and} \quad \beta/\tau = \frac{(k-1)^2}{k^2 \tau} \tag{60}
$$

and of course

$$\frac{\beta}{\alpha\tau} = \frac{(k - 1)^2}{(k^2 - 1)\tau} \tag{61}$$

(Note that when β/τ is computed recursively, as in equation (59), frame time, τ, is contained implicitly in the righthand term. However, when the steady state β/τ is computed, τ is contained explicitly.)

Exactly the same recursion formulas, equations (51) through (61), apply to position at an airborne unit in terms of x and $\dot{x}$, and y and $\dot{y}$. However, at a ground unit there is some simplification as there is no velocity term. In this case, the recursion formulas appear for x (or y) as

$$C^{-1}(n+1/n+1) = \frac{1}{k} C^{-1}(n/n) + 1/\sigma^2_{3x} \tag{62}$$

$$S(n+1/n+1) = S(n/n) + \frac{1}{\sigma^2_{3x}} C(n+1/n+1)\cdot DX \tag{63}$$

and the effective smoothing constants are similar to equation (57).

In the steady state condition for position adjustment at a ground unit, k is related to the smoothing parameter alpha, by

$$\alpha = (k - 1) / k \tag{64}$$

At initialization, and after a change in task (unit type), values of k and the associated steady state alpha and beta will be established. That is, equations (60) and (64) represent pre-computed, stored smoothing constants.

The recursive operation of updating (C(n/n) requires no matrix inversion. It proceeds as follows, inserting the appropriate variance, σ^2:

$$\text{define } C(n/n) = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \tag{65}$$

$$\text{then } C(n+1/n+1) = \begin{bmatrix} a' & b' \\ b' & c' \end{bmatrix} \tag{66}$$

where

If solution:	If no solution:
$\delta = 1 + k(a + 2\tau b + \tau^2 c)/\sigma^2$	$\delta = 1$
$a' = k(a + 2\tau b + \tau^2 c)/\delta$	$a' = k(a + 2\tau b + \tau^2 c)$
$b' = k(b + \tau c)/\delta$	$b' = k(b + \tau c)$
$c' = k(c + k\{ac - b^2\}/\sigma^2)/\delta$	$c' = kc$

(67)

Then the smoothing constants become

$$\alpha = a'/\sigma^2 \qquad \alpha = (k^2 - 1)/k^2$$

$$\beta/\tau = b'/\sigma^2 \qquad \beta/\tau = (k - 1)^2/k^2\tau \tag{68}$$

and of course,

$$\frac{\beta}{\alpha\tau} = b'/a' \tag{69}$$

For a ground unit, the position covariance is updated simply

$$ax' = \frac{\sigma_{3x}^2 \cdot k \cdot ax}{\sigma_{3x}^2 + k \cdot ax} \quad \text{and} \quad ay' = \frac{\sigma_{3y}^2 \cdot k \cdot ay}{\sigma_{3y}^2 + k \cdot ay} \tag{70}$$

and the smoothing constants become

$$\alpha_x = ax'/\sigma_{3x}^2 \quad \text{and} \quad \alpha_y = ay'/\sigma_{3y}^2 \tag{71}$$

The variances σ_{3x}^2 and σ_{3y}^2 above refer to normal mode operation. Of course, in the ranging mode these would be replaced by σ_{2x}^2 and σ_{2y}^2.

ADJUSTMENT OF STATE

The adjustments to estimates of state, which is the application of equation (55) or (63) to obtain S(n+1/n+1), will be shown in detail. Determination of correction factors Δt, Δd, Δx, Δy, $\Delta \dot{x}$ and $\Delta \dot{y}$ during the start-up process has been indicated in equation (48) or (50), or logical extension thereof. The remaining correction factors are

$$\Delta t = \alpha \cdot DR - \alpha^* \cdot TF$$

(α^*, the appropriate steady state smoothing constant is always applied to feedback) (72)

$$\Delta d = \frac{\beta}{\alpha} \cdot \Delta t / \tau$$

(A reference site sets TF to zero)

Then for all non-master units,

$$\dot{t}_{n+1} = \dot{t}_n + \Delta d$$

$$t_{n+1} = t_n + \Delta t + \tau \cdot \dot{t}_{n+1} \qquad (73)$$

are the updated estimates of system time.

Position adjustment factors, presently computed for ground and airborne units, at end of frame, are

$$\Delta X = -\alpha \cdot DX + \alpha^* \cdot XF$$

$$\Delta Y = -\alpha \cdot DY + \alpha^* \cdot YF \qquad (74)$$

(as with time, the appropriate steady state smoothing constant, α^* is always used with feedback)

At a ground unit these become directly

$$\Delta\emptyset = \Delta Y$$
$$\Delta\lambda = \Delta X \qquad (75)$$

which will be entered into the position update equations.

At an airborne unit, which performs position update every frame, the factors at the end of frame are

$$\Delta\emptyset = Vy\cdot\tau + \Delta\dot{y}\cdot\tau + \Delta Y$$
$$\Delta\lambda = Vx\cdot\tau + \Delta\dot{x}\cdot\tau + \Delta X \qquad (76)$$

while at any other time, they are simply

$$\Delta\emptyset = Vy\cdot\tau$$
$$\Delta\lambda = Vx\cdot\tau \qquad (77)$$

Then both ground and airborne units determine new positions

$$\emptyset_{n+1} = \emptyset_n + \left[\Delta\emptyset - \frac{\Delta\lambda^2}{2Nr_n}\cdot\tan\emptyset_n\right]/Rm_n \qquad (78)$$

(The terms in brackets appear in the parameter subroutine of Section V as

$$[\Delta\emptyset - \Delta\lambda^2\cdot TAN].)$$

$$\lambda_{n+1} = \lambda_n + \Delta\lambda/(Nr_{n+1}\cdot\cos_{n+1}) \qquad (79)$$

(The term Nr appears as 1/(A5·A1) in the parameter subroutine of Section V.)

UNCERTAINTY MATRIX

After the three 2 x 2 covariance matrices C_t, C_x and C_y have been established or updated, the output uncertainty matrix, or C matrix, first identified in equations (1) and (2), is computed.

At the master station, all six elements are zero.

At ground units in ranging mode and at airborne and reference units, the corner element, stt is just

$$\text{stt} = \text{at} \tag{80}$$

while at a ground unit in normal mode the effect of position error is included by

$$\text{stt} = \text{at} + (\text{sxx}\cdot\text{CX2} + \text{syy}\cdot\text{CY2} + 2\text{sxy}\cdot\text{cx}\cdot\text{cy})/N^2 \tag{81}$$

At a reference site, this is the only element computed, all others being set to zero.

At ground and airborne units, the following relations correlate time and position uncertainty.

Ranging mode:

$$\begin{aligned} \text{stx} &= 0 \\ \text{sty} &= 0 \end{aligned} \tag{82}$$

Normal mode, with time solution

$$\begin{aligned} \text{stx} &= \text{stt}\cdot\text{E2/E1} \\ \text{sty} &= \text{stt}\cdot\text{E3/E1} \end{aligned} \tag{83}$$

Normal mode, without time solution

$$\begin{aligned} \text{stx} &= k_1\cdot\text{stx} \\ \text{sty} &= k_1\cdot\text{sty} \end{aligned} \tag{84}$$

where the appropriate (ground or airborne) value of k_1 is used.

The diagonal elements, sxx and syy for a ground unit are the corner elements of C_x and C_y. That is:

$$\left.\begin{aligned} \text{sxx} &= \text{ax} \\ \text{syy} &= \text{ay} \end{aligned}\right\} \text{(ground)} \tag{85}$$

While for airborne units, they are

$$\begin{aligned} sxx &= (stx)^2/at + ax \\ syy &= (sty)^2/at + ay \end{aligned} \tag{86}$$

For an airborne unit in ranging mode, the above reduces to equation (85) since stx and sty are zero. In normal mode, however, the time dependence of the position determination is illustrated.

The remaining term, sxy, correlating the x and y position uncertainty, has four possible forms:

No solution

$$sxy = k_2 \cdot sxy \tag{87}$$

Solution obtained, ranging mode

$$sxy = -kp_2 \cdot cxy/E1 \tag{88}$$

Solution obtained, normal mode

$$\begin{aligned} sxy &= kp_2 \cdot X3/\nabla & \text{(ground)} \\ sxy &= stx \cdot sty/at - kp_2 \cdot cxy/E1 & \text{(airborne)} \end{aligned} \tag{89}$$

where again, the appropriate parameters k_2 and kp_2 are entered into equations (87) to (89)

The diagonal elements stt, sxx and syy, it can be seen, are obtained directly from the updating process for the three separate 2 x 2 covariance matrices. This process considers whether or not a solution has occurred and the relationship of the solution variance to the current or tracked variance. Compromise with convenience, however, has been utilized in determining the cross terms, or off-diagonal elements. In this case, if a solution occurs, the uncertainty is taken to be the steady state reduction of variance (a function of the discount factor $k(k_t$ or $k_p))$ times the solution variance. While if no solution occurs, the uncertainty from the previous frame is increased by the discount factor k.

SECTION III

INTERRUPTS

GENERAL

Once the start action has been taken by the operator, the sequencing of the program is under the control of interrupts. External interrupts initiate the processing of received messages, and internal interrupts (controlled by computer clock) initiate end of frame processing and transfer messages to the radio for transmission. The following paragraphs provide a brief sketch of these operations.

PROGRAM START

Program start is divided into two phases -- Data Entry and Initialization. During the data entry phase, the program is running, but not cycling in the sense of position and time extrapolation. Data is entered and response to certain commands is permitted. Under normal circumstances the minimum read-in set consists of unit ID, slot assignment(s), position (known or initial assumption), velocity (airborne units), unit task designation and mode of operation.

The initialization phase is begun by operator action. The action causes the initialization routine to be entered, which determines constants and starts cycling and extrapolation. The computer clock is read and the time for the first end of frame processing interrupt is determined. At the MASTER unit this leads to the first position report transmission. At the other units this begins the listening process.

MESSAGE RECEIVED INTERRUPT

The message received interrupt initiates a two-stage process. The first stage is essentially a housekeeping process, kept deliberately as short as possible, which accomplishes little more than picking up and stacking the message for further processing. The second stage is a filtering and an information extraction process. In this paper, only those parts of the process that pertain to the requirements for recursive navigation will be addressed in detail.

Whatever routine is in process when a message received interrupt occurs, operation of that routine is temporarily suspended while the first stage, message preprocessing, is accomplished. Operation is then resumed on the interrupted routine. It is conceivable that with certain long routines, more than one interrupt could occur, thus building up a backlog of messages stacked for future processing. When operation of the interrupted routine is finally concluded, and any other priorities satisfied, the second stage of the message processing is begun. This is the filtering and information extraction process, which may, in turn, be temporarily suspended by other interrupts.

TIME TO TRANSMIT INTERRUPT

Whatever routine or subroutine is in process, when a time to transmit interrupt occurs, operation of that routine is temporarily suspended while a previously constructed message is conveyed to the transmitter for transmission at a predetermined time. Operation of the interrupted routine is then resumed.

The time to transmit interrupt is associated with a reading on the computer clock. The actual time to transmit is associated with a reading on the radio clock.

Proper management and control functions must be accomplished so that transmit and receive operations will not conflict.

TIME TO BEGIN END OF FRAME PROCESSING INTERRUPT

The time to begin end of frame processing interrupt is associated with a reading on the computer clock. It is set to occur so as to provide adequate time for end of frame processing prior to the transmission of the periodic position report. This will not interrupt routines entered due to message received or time to transmit interrupt, but will allow them to be completed. It will, however, prevent any further second stage message processing until after completion of the end of frame processing.

The time for this interrupt is adjusted each frame (if messages were received or transmitted) to keep the radio clock and computer clock in rough synchronization.

SECTION IV

ROUTINES

DISCUSSION

There are three major routines that are entered in response to external or internal interrupts. These are the initialization, message processing, and end of frame processing routines. In addition to accomplishing the recursive navigation functions of position location and time synchronization, these routines maintain control of the program by setting and resetting counters, indices and future interrupts. Additionally, there is an enter data phase. This phase occurs prior to the start of the recursive position location and time synchronization functions.

The computational burden of the least squares linear unbiased estimate of time and position error is distributed between the message processing routine and the end of frame processing routine. The former determines the measurement, computes weighting factors and maintains running sums. The latter utilizes the sums to estimate time and position errors; these estimates in turn are entered into difference equations (smoothing equations) to arrive at adjustments to be made to current estimates of time, position (and velocity), followed by a position extrapolation.

The logical design of these routines is presented in the following pages. To make the logic flow as compact as possible, certain parts of these routines have been separated out as subroutines. The section on subroutines follows this section, which in turn is followed by a complete alphabetic parameter listing.

ENTER DATA (PHASE)

This is not a routine, but a necessary phase of the start-up process. The computer program is, of course, running in the sense of responding to commands (interrupts) and looking for things to do. But the navigation and timing functions of position extrapolation and synchronization have not yet begun. Before the start action is taken, certain switch settings must be accomplished and parameter values entered.

1. The usual read-in sets consist of,

ϕo	latitude	in minutes north of ϕr
λo	longitude	in minutes west of λr
Zo	altitude	in feet above sea level

AIRBORNE: initial velocity estimate

Vo speed	in knots
Ho heading	in degrees from true north

Unit identification, ID

Slot assignment, Sa, for the periodic position report.

The above seven parameters are programmed zero. Some, if not entered will be estimated by the program; others, remaining zero will prevent the start-up process.

Unit identification must be entered at all units before start-up.

Initial position must be entered at MASTER or REFERENCE sites prior to start-up.

Any non-MASTER unit not entering slot assignment, Sa, is forced to remain in the QUIET mode. (The assignment may be received by message interpretation.)

At a GROUND or AIRBORNE unit initial position, velocity and wind data (as applicable) may be entered before start-up. The values entered being the best estimate of conditions at the time the start action is taken. If not entered, ϕc, λc, Zc (ground) or ϕc, λc, Za (airborne) is supplied by the program. And, if velocity and winds are not supplied, the program starts with estimates set to zero.

2. Switch settings

$$\text{Unit Task Designation} \begin{cases} \text{MASTER} \\ \text{REFERENCE} \\ \text{GROUND} \\ \text{AIRBORNE} \end{cases}$$

$$\text{Mode of Operation} \begin{cases} \text{UNSPECIFIED} \\ \text{RANGING} \\ \text{NORMAL} \\ \text{QUIET} \end{cases}$$

The mode of operation will usually be left on UNSPECIFIED. Unit designation must be made before start-up. It may be changed during operation, manually, in which case, the program will re-enter the initialization routine.

3. Any additional parameters, as desired, may be read in, provided coordination among all units is accomplished.

INITIALIZATION (ROUTINE)

The initialization routine has two entry points. When the operator initiates the start action, the routine is entered at point 1. Initial parameter values are computed and the process of cycling and position extrapolation is begun. (Item 3. actually represents precomputed and stored program constants.) The routine is also entered when message processing or data entry causes a change in unit designation. In this case, the routine is entered at point 10. and the appropriate adjustments made.

When data entry is complete, the start action initiates the cycling process. The initialization routine proceeds as follows:

1. Test identification entry.

   If ID = 0 (no entry), hold up start action, alert operator.

   If ID has been entered, proceed.

2. AIRBORNE: Read computer clock,

   set to = computer clock reading.

3. Fixed constants.

$a = SMA$	convert units
$A1 = 1/a$	inverse
$f = 1/FLAT$	flattening
$e = f(2 - f)$	eccentricity squared
$A2 = a(1 - e)$	
$A3 = 1/(a \cdot \sqrt{1 - e})$	

Set $\emptyset = (\emptyset c + \emptyset r)$ sector center latitude

Get parameters (subroutine)

$DTERM = 3 \cdot e \cdot Rm \cdot \sin \emptyset \cdot \cos \emptyset / 2A4$

$Nc = a/A5$ mean normal radius

$C1 = c/INDEX$

$C1inv = 1/C1$

4. Test position entry.

MASTER or REFERENCE:

If no position entry, hold up start action, alert operator.

If position available, proceed. (i.e., $\emptyset o + \lambda o + Zo \neq 0$).

GROUND or AIRBORNE:

If no initial position is given, set

$\emptyset o = \emptyset c$

$\lambda o = \lambda c$

GROUND: $Zo = Zc$
AIRBORNE: $Zo = Za$

If initial position available, proceed.

5. Set initial position.

$\emptyset = \emptyset o + \emptyset r$

$\lambda = \lambda o + \lambda r$

$Z = Zo$

NON-AIRBORNE: set updated position

$\emptyset' = \emptyset$

$\lambda' = \lambda$

6. Initial clock parameter values

$\dot{t}$ = 1 (if master establishing one second frame)

$\dot{t}$ = .993 (If master synchronizing to east coast Loran chain)

$T2 = \tau \cdot \dot{t}$ frame duration

$T1 = T2/\#$ slot duration

Get parameters (subroutine)

7. AIRBORNE: Set velocity

If $Vi = 0$ (unavailable), set $V = Vo$ (manual input)

If $Vi \neq 0$ (available), set $V = Vi$ (instrument input)
Then $Va = V$ assumed velocity

If $Hi < 0$ (unavailable), set $H = Ho$ (manual input)

If $Hi \geq 0$ (available), set $H = Hi$ (instrument input)
Then $Ha = H$ assumed heading

Check for manual entry of wind data, Vw and Hw (wind speed and direction from) and convert to wind estimate components

$$Wx = Vw \cdot \sin Hw$$

$$Wy = -Vw \cdot \cos Hw$$

Then the x, y velocity components are computed

$$Vx = Wx - V \cdot \sin H \qquad \text{speed west}$$

$$Vy = Wy + V \cdot \cos H \qquad \text{speed north}$$

compute angular values

$$V\phi = Vy/Rm$$

$$V\lambda = Vx \cdot A1 \cdot A5/\cos\phi$$

8a. GROUND: zero velocity

$$Vx = Vy = 0$$

8b. ALL Units:

Set State = UU

Operating Mode = QUIET

9. Read computer clock

Set tc = current computer clock reading

S = 0	dummy slot assignment
t = 0	dummy transmit time

Set end of frame interrupt time

Teoc = tc − δp + T2

Compute value of, <u>but do not set</u>, transmission interrupt time

Tip = tc − δB

AIRBORNE: extrapolate

txp = tc − to	time interval from start action
$\emptyset = \emptyset + V\emptyset \cdot txp$	latitude
$\lambda = \lambda + V\lambda \cdot txp$	longitude

10. Set operating mode, status, smoothing constants and task sensitive constants.

		AIR	GROUND	REF	MASTER
k_1	time discount factor	kta	ktg	ktr	ktl
k_2	x-pos " "	kpa	kpg	kpg	kpg
k_3	y-pos " "	kpa	kpg	kpg	kpg
kp_1	steady state α_t	ktal	ktgl	ktrl	ktll
kp_2	" " α_x	kpal	kpgl	kpgl	kpgl
kp_3	" " α_y	kpal	kpgl	kpgl	kpgl

		AIR	GROUND	REF	MASTER
kv_1	steady state β/α_t	kda	kdg	kdr	kdl
kv_2	" " β/α_x	kva	0	0	0
kv_3	" " β/α_y	kva	0	0	0
CTS	frames to Sync	CTSG	CTSG	CTSR	CTSR
CTU	frames to Unsync	CTUG	CTUG	CTUR	CTUR
state				_P	SP
operating mode					NORMAL
MIS (external)	master-in-sight indicators	0	0	1	1
MIS (internal)		0	0	0	0
RIS (internal)	ref-in-sight indicator	0	0	0	0

11. Set Counters

    CP, CT = 0

12. Zero sample counts and feedback

    NS, N, NT, NSP, NSU, NUP, TF, XF, YF = 0

13. Zero running sums

    d, dx, dy, dt, cx, cy, cxy, CX2, CY2 = 0

14. Clock advance

    IND = 0 (off) — in data indicator

    $\Delta C = 0$ — change in clock advance

MESSAGE PROCESSING

Message processing is a two stage process initiated by a message received interrupt. The first stage -- preprocessing -- is essentially a housekeeping process, kept deliberately as short as possible. The second stage -- processing -- is a filtering and information extraction process, which may be long or short, and which may cause future actions to take place. Those portions of the message processing routines which are required to support recursive navigation are discussed in the following sections.

Preprocessing

The preprocessing routine is structured so that on each message received or message transmitted interrupt, a correspondence between the radio clock reading and the computer clock reading can be established.

Upon receipt of message received or message transmitted interrupt:

Set trad = tj

where tj is the clock reading inserted into the received message by the receiver, or the transmit time requested for the transmitted message.

Cause the computer clock to be read, and stored.

Set tcomp = tc

where tc is the current computer clock reading (in computer clock units)

Set IND = 1

Every message is saved, or stacked, for future processing.

Processing

Message processing is a combination filtering and action/no-action process. Many tests may be performed on the message before deciding what to do with it.

1. Compute time in transit. (all messages)

$$Ts = [tj - \{t \underset{\text{modulo } \tau}{+} T1\ (Sj \overset{\text{modulo \#}}{-} S)\}] - \delta \qquad \text{time in transit}$$

where t and S are the time and slot number of the local unit's last position report, and tj and Sj are the arrival time and slot number of the current message being processed. T1 is local slot duration and δ accounts for internal delays. If the message has been held in stack during end of cycle processing it is necessary to account for any change in time rate estimate by setting

$$Ts = Ts + T2$$

2. Justify. The transit time, Ts, must be kept in the range $-0.005 \leq Ts < 0.995$ seconds.

If $Ts < -0.005$ seconds,

Set $Ts = Ts + 1.0$ seconds.

If $TS \geq 0.995$ seconds,

Set $Ts = Ts - 1.0$ seconds.

3. Position Report: if so labeled, a message containing the position $Pj = (\emptyset j, \lambda j, Zj)$, which arrived at radio time tj in time slot Sj is to be processed.

a. Enter data into the Data Source Table, indexed by unit ID:

IDj	identity
tj, Sj	arrival time, slot number
state_j, task_j	state, task designation
$\emptyset j$, λj, Zj	position
Vj, Hj	velocity
sttj, stxj, styj sxxj, sxyj, syyj	uncertainty matrix

b. If Task_j is MASTER, set MIS = 1 (on) master in sight.

If Task_j is REFERENCE, set RIS = 1 (on) reference in sight.

c. REFERENCE:

If Task_j is not MASTER, <u>and</u> if MIS = 1, then exit.

If Task_j is GROUND or AIRBORNE, <u>and</u> if RIS = 1, then exit.

d. Check source state

If state_j is UU, exit to main program

If state_j is UP,

set NUP = NUP + 1, then exit to main program

If state_j is SU

set NSU = NSU + 1

If CT $\neq$ 0, or if local unit is MASTER, exit to main program

If state_j is SP,

Test received MIS indicator

If MIS_j = 0, exit to main program.

NSP = NSP + 1

If NSP $\leq$ 3, store IDj in SIDj (indexed by NSP)

e. If operating mode ranging, skip to item j.

f. AIRBORNE: update position.

$txp = tj - t \quad \text{modulo } \tau$ extrapolation interval

If $txp > (\tau - \delta p)$, set $txp = txp - \tau$

$\emptyset' = \emptyset + V\emptyset \cdot txp$

$\lambda' = \lambda + V\lambda \cdot txp$

g. Use geometry subroutine to find computed range, Rc, and direction cosines, SIN Ψ and COS Ψ. Upon return from geometry, enter Rcj into the data source library.

h. The measured range is

$$Rt = \nu p \cdot Ts$$

(the value of νp depends upon whether Loran data or UHF data is being processed)

i. Proceed to Summation subroutine.

j. Feedback: If this is a position report from the Master it contains the time and position feedback adjustments.

a. NON-MASTER: set

$$TF = TFj$$

$$XF = XFj$$

$$YF = YFj$$

If any one, of $|TF| > DM4$, or $|XF| > 1/2$ n.m., or $|YF| > 1/2$ n.m. set all, $TF = XF = YF = 0$.

(where the j indicates data in the message to be stored.)

4. Ranging Reply addressed to local unit: if so labeled and addressed, a message containing the position $Pj = (\emptyset j, \lambda j, Zj)$, and the apparent one way transmission time, Tsr, as measured by the responder, which arrived at time tr in slot Sr, in response to interrogation, is to be processed. The position, Pj, represents the mean position over the interrogation interval. While the quantities tr and Sr represent the same physical data as would tj and Sj when processing a position report, the distinction is made to avoid confusion when both tr and tj must be used, in which case, tr refers to the current reply message in process, and tj refers to the last position report received from the same unit.

a. Compute the factor

$tq = (tr - ti)/2$, subtraction modulo τ

(where ti was the time of interrogation)

b. AIRBORNE: extrapolate position

$$txp = tq + T1\ \overline{(si - S)}^{\text{modulo } \#}$$

$$\emptyset' = \emptyset + V\emptyset \cdot txp$$

$$\lambda' = \lambda + V\lambda \cdot txp$$

(in the previous, ti and Si are the time and slot number of the interrogation sent out by local unit. They are in the interrogation table).

c. Use geometry subroutine to find computed range, Rc, and direction cosines SIN Ψ and COS Ψ. Upon return from geometry, store Rcj in data source library.

d. The measured range is

$Rt = \nu p\ (Tsr + Ts)/2$ measured range

e. If reply synchronized (time state S); compute

$$T' = \frac{Tsr + Ts}{2} \qquad \text{one way time}$$

If either source or receiver is AIRBORNE, compute modification to time measurement to account for relative velocity:

$$T' = T' + \frac{tq}{C}\left[(Vx + Vj \cdot \sin Hj)\mathrm{SIN}\Psi - (Vy - Vj \cdot \cos Hj)\ \mathrm{COS}\Psi\right]$$

where Vj and Hj come from the data source library.

$dt = dt + Ts - T'$

$NT = NT + 1$

f. If reply position state_j is P (known), then

proceed to summation subroutine. Then exit.

Otherwise, exit.

5. Ranging Interrogation addressed to local unit: if so labeled and addressed, a request for a timing measurement and a reply has been received at time tj in slot Sj. The request asks for a reply in slot Sr to unit IDj, containing the time measurement Tsr and the mean position over the interrogation interval.

a. The transmission time, on radio clock, is

$$tr = t + Tl[(Sr - Sj) + (Sj - S)]$$

modulo τ modulo #

(where t and S refer to time and slot number of local unit's last position report, tj and Sj the arrival time and slot of the current message--the interrogation, and Sr the requested reply slot.)

b. AIRBORNE: determine position for transmission

$$txp = (tj - t) + (tr - tj)/2$$

modulo τ

$$\varnothing t = \varnothing t + V\varnothing \cdot txp$$

$$\lambda t = \lambda t + V\lambda \cdot txp$$

c. Prepare message for transmission at time tr, which is the start of slot Sr, as per request, containing Tsr, which is the Ts just measured, Øt, λt, Z, and current state, as well as own (sender) ID, slot number Sr, message type, and the addressee identification IDr.

Set time to transmit interrupt on the computer clock to

$$Trr = Tip + Tl[(Sr - Sj) + (Sj - S)]$$

modulo #

d. Exit.

END OF FRAME PROCESSING (ROUTINE)

The end of frame processing routine is entered once per frame in response to a timed interrupt. The time for this interrupt--the time to begin end of frame processing--is computed or adjusted at least once per frame to assure that the processing is completed prior to the time for the position report transmission. This routine operates on data gathered during the frame to determine adjustments to estimates in timing, position and velocity; to consider changes in data status or mode of operation; and to reset certain parameters for another frame of operation.

0. Internal Synchronization

If IND = 1, compute change in clock advance,

$$\Delta C = [\text{tcomp} \overset{\text{modulo } \tau}{-} (\text{Tip} + B)] - (\text{trad} \overset{\text{modulo } \tau}{-} t)$$

If $\Delta C < -0.02$ seconds, set $\Delta C = \Delta C + 1.0$ seconds

If $\Delta C \geq +0.98$ seconds, set $\Delta C = \Delta C - 1.0$ seconds.

If now, $\Delta C > 0.5$, set $t = t + T2$

Set IND = 0

0a. Compute mean range error

MRE = d·Clinv/N

1. MASTER: compute feedback

Set TF = XF = YF = 0 zero out feedback

If $N \geq 2$, compute feedback

TF = ktm · d · Clinv/N, limiting $|TF| \leq$ DM4

Get position adjustment (2 x 2) (subroutine)

XF = kpm · DX, limiting $|XF| \leq 1/2$ n.m.

YF = kpm · DY, limiting $|YF| \leq 1/2$ n.m.

$t = t + T2$ transmit time

(If the master unit is synchronizing to the Loran net, then it will proceed with that Loran data, much as a reference unit with items 2., 3., 6., and 10. The details will be omitted.)

Then skip to item 12.

2. Set Δt and $\Delta d = 0$.

Set TN and PN = 0.

If CT = 0, 1 or 2; or if CT $\leq$ -CTU, then proceed with Gross Sync Phase:

If $n \neq 0$, compute

Δt = MRE
TN = 1
Restore clock rate (T2)
Set items SM(t) and TI(t) to zero, and
Advance Time Phase to Start.

Skip to item 6, to compute times.

3. REFERENCE: Compute time estimate adjustments

Set PN = 1	position known
Set DR, Ds, $\sigma SQ_1 = 0$	zero parameters

Set i = 1 (reset index for system time)

If operating mode is RANGING, then

If NT $\neq$ 0, set

TN = 1	time adjustment computed
DR = dt/NT	time error estimate
σSQ_1 = 1/2NT	solution variance

If operating mode is NORMAL or QUIET, then

If SYNC MODE is TOA, and if N $\neq$ 0, then

TN = 1	time adjustment computed
DR = MRE	time error estimate
σSQ_1 = 1/N	solution variance

If time state = S, test magnitude of DR

If $|DR|$ > DM5, set TN = 0, MIS = 0 (external only)

If $|DR|$ > DM4, limit magnitude of DR to DM4

If phase 2 (start up--system clock), then

Set Ds = DR•Cl — range error estimate

Call Start (subroutine)

If Δs is non-zero, set

$\Delta t = \Delta s \cdot Clinv$	system time adjustment
$\Delta d = \Delta \dot{s} \cdot Clinv$	" " rate adjustment
$stt = a_i$	time uncertainty
Skip to item 6. (to compute times)	

If phase 3 (recursive), then

Call Update (subroutine)

$\Delta t = \alpha_i \cdot DR$	system time adjustment
$stt = a_i$	time uncertainty
Skip to item 6. (to compute time and time rates)	

(Note: at a reference site, uncertainty factors stx, sty, sxx, sxy, and syy are always zero.)

4. AIRBORNE: set $\Delta\dot{x}$, $\Delta\dot{y}$ = 0. — zero velocity adjustment

5. GROUND or AIRBORNE: set ΔX, ΔY = 0. — zero position adjustment

Set DR, DX, DY, σSQ_1, σSQ_2, σSQ_3, = 0 — zero parameters

If state not SP, then set TF, XF, YF = 0 — zero feedback.

If operating mode RANGING, and

If NT $\neq$ 0, then set

TN = 1	time adjustment computed
DR = dt/NT	time error estimate
σSQ_1 = 1/2NT	solution variance

If NS $\geq$ 2, then set PN = 1 — position adjustment computed

If NS < 2, and if GROUND unit with position state P, then

Set PN = 1 — assume position O.K.

Skip to 5b. (to limit DR and obtain time adjustment).

If operating mode NORMAL or QUIET, and

If NS < 3, and if GROUND unit with position state P, then

Set PN = 1 — assume position O.K.

If $N \neq 0$, set

TN	= 1	adjustment computed
DR	= MRE	time error estimate
σSQ_1	= 1/N	solution variance

Skip to 5a.

If NS = 2, and if AIRBORNE unit, then

If time state S,

Set PN = 1 — adjustment (to be) computed

Skip to 5b.

If NS $\geq$ 3, then

Set TN and PN = 1 — position and time adjustments computed

Get position and time adjustment (3 x 3)

If GROUND unit with position state P, then set

DR	= MRE	time error estimate
σSQ_1	= 1/N	solution variance
TN	= 1	adjustment computed

Skip to 5a.

If TN = 0 (no solution), skip to 5b

$\sigma SQ_1 = E1/\nabla$ — solution variance

5a. If state SP, and if MIS = 1, then

GROUND: set SIGN = +1

AIRBORNE:

If MRE > MQ1, and DR < -MQ2, or
MRE < -MQ1, and DR > MQ2, then

SIGN = -1 significant disagreement
Else SIGN = +1

Both GROUND and AIRBORNE:

If TF > MQ2, and DR > 0, then

If SIGN = -1, set DR = MRE
Else set DR = 0

If TF < -MQ2, and DR < 0, then

If SIGN = -1, set DR = MRE
Else set DR = 0

5b. Test and Limit Magnitudes

If time state S, then

If $|MRE|$ > DM5, then

Set TN and MIS = 0.

Limit the magnitude of DR to DM4

Set Index i = 1.

If phase 2 (start up--system time), then

Set Ds = DR·C1

Call Start (subroutine)

If Δs is non-zero, set

$\Delta t = \Delta s \cdot C1inv$
$\Delta d = \Delta \dot{s} \cdot C1inv$
$stt = a_i$

Skip to 5c.

If phase 3 (recursive--system time), then

Call Update (subroutine)

$\Delta t = \alpha_i \cdot DR - kp_1 \cdot TF$

If operating mode is Ranging, then

stx, sty = 0

If $\sigma SQ_1 = 0$, then

$stx = k_i \cdot stx$

$sty = k_i \cdot sty$

time factor x $= k_i \cdot$time factor x

time factor y $= k_i \cdot$time factor y

time factor xy $= k_i \cdot$time factor xy

AIRBORNE units:

$stt = a_i$

If $\sigma SQ_1 \neq 0$, then

$stx = stt \cdot E2/E1$
$sty = stt \cdot E3/E1$

If operating mode $\neq$ Ranging, then

time factor x = $a_i \cdot E2^2/E1^2$

time factor y = $a_i \cdot E3^2/E1^2$

time factor xy = $a_i \cdot E2 \cdot E3/E1^2$

If operating mode = Ranging, then

time factor x = 0
time factor y = 0
time factor xy = 0

Skip to 5d.

GROUND units:

stt = a_i + pos factor
stx = stt•pos factor x
sty = stt•pos factor y
time factor x = 0
time factor y = 0
time factor xy = 0

5c. Independent or Hyperbolic position solution.

(Used by GROUND units, any state, and by AIRBORNE units with time state U.)

If PN $\neq$ 0, and NS $\geq$ 3, then

σSQ_2 = X2/∇ solution variance, x

σSQ_3 = Y3/∇ solution variance, y

Skip to 5e.

5d. Dependent or Direct Ranging position solution.

(Used by any unit in ranging mode, and by AIRBORNE units with time state S in NORMAL or QUIET.)

If NS $\geq$ 2, then

Get position adjustment (2 x 2) (subroutine)

If PN $\neq$ 0, then

σSQ_2 = CY2/E1 solution variance, x

σSQ_3 = CX2/E1 solution variance, y

5e. Limit position adjustments

If position state U, then

Limit the magnitudes of DX and DY to 30 n.m.

If position state P, then

Limit the magnitudes of DX and DY to QMAX.

5f. Set Index $i = 2$.

If Phase 2 (start up--position), then

Set Ds = -DX

Call Start (subroutine)

If Δs is non-zero, set

$$\Delta x = \Delta s$$
$$\Delta \dot{x} = \Delta \dot{s}$$
$$sxx = a_i$$

Set Index $i = 3$

Set Ds = -DY

Call Start (subroutine)

If Δs is non-zero, set

$$\Delta y \quad \Delta s$$
$$\Delta \dot{y} = \Delta \dot{s}$$
$$syy = a_i$$
$$stx,\ sty,\ sxy = 0$$

Skip to item 6.

If phase 3 (recursive--position)

AIRBORNE units:

Call Update (subroutine)
$sxx = a_i +$ time factor x

Set Index $i = 3$
Call Update (subroutine)
$syy = a_i +$ time factor y

If operating mode NORMAL or QUIET, and if PN = 1, set

$sxy =$ time factor xy $- kp_2 \cdot cxy/E1$

GROUND units:

Call Update Ground (subroutine)
$sxx = a_i$ + time factor x

Set Index i = 3
Call Update Ground (subroutine)

$syy = a_i$ + time factor y

If operating mode NORMAL or QUIET, and if $\sigma SQ_1 \neq 0$, then

$sxy = kp_2\ X3/\nabla$
pos factor = $(a_2 \cdot CX2 + a_3 \cdot CY2 + 2 \cdot sxy \cdot cx \cdot cy)/N^2$
pos factor x = E2/E1
pos factor y = E3/E1

If operating mode Ranging, then

pos factor = 0
pos factor x = 0
pos factor y = 0

Both AIRBORNE and GROUND units:

$\Delta X = -\alpha_2 \cdot DX + kp_2 \cdot XF$ position adjustment, x

$\Delta Y = -\alpha_3 \cdot DY + kp_2 \cdot YF$ position adjustment, y

If $\sigma SQ_2 \neq 0$, and

If operating mode Ranging, then

$sxy = -kp_2 \cdot cxy/E1$

If $\sigma SQ_2 = 0$, then $sxy = k_2 \cdot sxy$

6. Adjust estimates of time rates

NON-MASTER: If phase 3 (recursive--system time), then

$\Delta d = \beta_1 \cdot \Delta t/\tau$

limit $|\Delta d| \leq$ DMAX

6a. All phases:

$\dot{t} = \dot{t} + \Delta d$ system time rate

$T2 = \tau \cdot \dot{t}$ nominal frame time

$T1 = T2/\#$ slot duration

$\tau f = T2 + \Delta t$ frame extrapolation

$t = t + \tau f$ position report transmit time

7. AIRBORNE: compute extrapolation velocity and adjust velocity factors.

$\Delta\dot{x} = \beta_2 \cdot \Delta x/\tau f$

limiting the magnitude of $\Delta\dot{x}$ to VMAX

$\Delta\dot{y} = \beta_3 \cdot \Delta y/\tau f$

limiting the magnitude of $\Delta\dot{y}$ to VMAX

$Wx = Wx + \Delta\dot{x}$ updated wind estimate, west

$Wy = Wy + \Delta\dot{y}$ updated wind estimate, north

If air data inputs available, get new speed and heading, but limit changes.

$\delta q = Vi - V$

limit the magnitude of δq to VMAX

$V = V + \delta q$

$\delta q = Hi - H$

limit the magnitude of δq to HMAX

$H = H + \delta q$

If no air data inputs, check for manual speed and heading entries.

$\delta q = Vo - V$

If $\delta q \neq 0$, set $V = Vo$

$\delta q = Ho - H$

If $\delta q \neq 0$, then set $H = Ho$

Look for current manual entry of wind data, Vw and Hw. If manual entry of wind data has been made this frame, then reset:

$Wx = Vw \cdot \sin Hw$

$Wy = -Vw \cdot \cos Hw$

Compute extrapolation velocity

$Ve = (Va + V)/2$

$He = (Ha + H)/2$
(being careful of the discontinuity at 360°)

$Vx = Wx - Ve \cdot \sin He$

$Vy = Wy + Ve \cdot \cos He$

8. GROUND or AIRBORNE: determine new position and compute new parameter values for use during next frame,

GROUND: set $\Delta\emptyset = \Delta y$

$\Delta\lambda = \Delta x$

AIRBORNE: set $\Delta\emptyset = Vy \cdot \tau + \Delta\dot{y} \cdot \tau + \Delta y$

$\Delta\lambda = Vx \cdot \tau + \Delta\dot{x} \cdot \tau + \Delta x$
(bearing in mind that Δx, Δy, $\Delta\dot{x}$, $\Delta\dot{y}$ will be non-zero only at end of frame in the recursive phase.)

$\emptyset = \emptyset + (\Delta\emptyset - \Delta\lambda^2 \cdot TAN)/Rm$

Get parameters (subroutine)

$\lambda = \lambda + \Delta\lambda \cdot A5 \cdot A1/\cos \emptyset$

If air data inputs available, set Z = Zi

If air data not available, set Z = Zo

GROUND: set $\emptyset' = \emptyset$

$\lambda' = \lambda$

9. AIRBORNE: determine new velocity components.

Va = V

Ha = H

Vx = Wx − Va·sin Ha

Vy = Wy + Va·cos Ha

V∅ = Vy/Rm

Vλ = Vx·A1·A5/cos ∅

10. State check

If TN = 1, then

If time state = S, set CT = -1.

If time state = U, then

If CT $\leq$ 0, set CT = 1

If CT > 0, set CT = CT + 1, and

If CT $\geq$ CTS, set time state = S, and
set CT = -1.

If TN = 0, then

If CT = +1, set CT = -1

If CT < 0, or if CT > 1, then

set CT = CT -1, and

If CT $\leq$ -CTU, then

set time state = U
(time phase will be gross sync next frame)

If position state = U, then

If PN = 1, set CP = CP + 1, and

If CP $\leq$ 0, set CP = +1

If CP $\geq$ CPK, set position state = P, and set CP = 0.

If PN = 0, set CP = CP -1, and

If CP $\leq$ -CPU, then set

CP = 0
position phase = start up
SM(x), SM(y) = 0
TI(x), TI(y) = 0.

If position state = P, then

If PN = 0, then

set CP = CP + 1

If CP $\geq$ CPU, and unit is AIRBORNE, then set

CP = 0
position phase = start up
position state = U
SM(x), SM(y) = 0
TI(x), TI(y) = 0.

If PN = 1, then set CP = 0.

11. NON-MASTER: mode check

If either TN or PN is zero, or if operating mode ≠ NORMAL, or if selected mode ≠ NORMAL, then go to mode check subroutine.

Otherwise, proceed without checking mode.

12. Zero sums, sample counts and feedback

Set d, dx, dy and dt to zero
Set cx, cy, cxy, CX2 and CY2 to zero
Set N, NS, NT, NSP, NSU, and NUP to zero
Set TF, XF and YF to zero

13. Prepare new position report for transmission at time t (radio clock), containing current state, current position, slot number, ID, task, MIS indicator, etc. (MIS indicator is always set to one in the outgoing message at MASTER and REFERENCE). The position loaded is,

$Pt = (\emptyset t, \lambda t, Z)$ where

$$\emptyset t = \emptyset - \emptyset r$$

$$\lambda t = \lambda - \lambda r$$

Then if NON-REFERENCE, set MIS = 0.

14. Check slot assignment

$\Delta S = (Sa - S)$ modulo #	forward slot adjustment
$S = Sa$	assigned slot

If $\Delta S \neq 0$, then

$t = t + Tl \cdot \Delta S$ modulo τ	position report time

adjust value of, <u>but do not set</u>, end of frame interrupt

$Teoc = Teoc + Tl \cdot \Delta S$

AIRBORNE: extrapolate

$txp = Tl \cdot \Delta S$	time interval
$\emptyset = \emptyset + V\emptyset \cdot txp$	latitude
$\lambda = \lambda + V\lambda \cdot txp$	longitude

Adjust value, <u>but do not set</u>, transmission interrupt

$Tip = Teoc + \delta p - \delta B + \Delta t + \Delta C$

15. Set Interrupts

If operating mode $\neq$ QUIET, set transmission interrupts

Tip (just computed)

If operating mode is RANGING, proceed to ranging interrogation subroutine.

15. Set Interrupts

If operating mode $\neq$ QUIET, set transmission interrupts

Tip (just computed)

If operating mode is RANGING, proceed to ranging interrogation subroutine.

Adjust value of, and set, end of frame interrupt

$Teoc = Teoc + \Delta t + T2 + \Delta C$

Set $\Delta C = 0$

SECTION V

SUBROUTINES

GENERAL

Much of the computation alluded to in the foregoing routines is now presented in detail in this section on subroutines.

The first, Parameters, supports both the initialization and the end of frame processing.

Geometry and Summation support message processing.

While the remaining seven, Position Adjustment 2 x 2, Position Adjustment 3 x 3, Start, Update, Update Ground, Mode Check and Ranging Interrogation Set-Up support the end of cycle processing.

PARAMETERS

This subroutine is used by the initialization routine and the end of cycle processing routine. It computes parameters which are a function of latitude: Rm, meridional radius of curvature, $\overline{\text{rinv}}$, inverse mean radius of curvature, and the factor TAN. It uses the local unit's latitude, $\emptyset$, and the constant e, (eccentricity squared) and the fixed constants, A2 and A3, computed in the initialization routine.

$$A4 = (1 - e \cdot \sin^2 \emptyset)$$

$$A5 = \sqrt{A4}$$

$$A6 = A5 \cdot A4$$

$$Rm = A2/A6$$

$$\overline{\text{rinv}} = A4 \cdot A3$$

$$TAN = 1/2 \cdot A5 \cdot A1 \cdot \tan \emptyset$$

GEOMETRY

For position reports, the geometry subroutine computes the straight line distance, Rc, between the local unit's updated position,

$P' = (\emptyset', \lambda', Z')$ and the position contained in message received, $Pj = (\emptyset j, \lambda j, Zj)$. It also computes the sine and cosine of the bearing angle (with respect to north) from P' to Pj. (Adapted from Reference 5.)

$$x = (\lambda' - \lambda r - \lambda j) \cdot Nc \cdot \cos(\emptyset j + \emptyset r)$$

$$\Delta\emptyset = \emptyset j + \emptyset r - \emptyset'$$

$$y = \Delta\emptyset \cdot Rm + x^2 \cdot TAN + \Delta\emptyset^2 \cdot DTERM$$

$$RS2 = x^2 + y^2$$

$$G1 = 1 + Z \cdot \overline{rinv}$$

$$G2 = 1 + Zj \cdot \overline{rinv}$$

$$Rc = \sqrt{G1 \cdot G2 \cdot RS2 + (Zj - Z)^2} \qquad \text{computed range}$$

$$Rcinv = 1/Rc$$

$$SIN\Psi = x \cdot Rcinv$$

$$COS\Psi = y \cdot Rcinv$$

For Loran inputs the geometry subroutine computes the surface range between the local unit's updated position, $\emptyset'$, λ', and the Loran transmitter, $\emptyset_T$, λ_T, using the Andoyer-Lambert formula.

$$W = \sin \emptyset_T \cdot \sin \emptyset' + \cos \emptyset_T \cdot \cos \emptyset' \cdot \cos (\lambda' - \lambda_T)$$

$$X1 = \text{arc cosine } W$$

$$Y1 = \sin X1$$

$$G3 = \sin \emptyset_T + \sin \emptyset'$$

$$G4 = \sin \emptyset_T - \sin \emptyset'$$

$$P1 = (3 \cdot Y1 - X1) \cdot K/(1 + W)$$
(Where $K = 0.5602164710 \times 10^{-5}$)

$$P2 = (3 \cdot Y1 + X1) \cdot K/(1 - W)$$

$$RC = (X1 + P1 \cdot G3^2 - P2 \cdot G4^2) \cdot 3443.93358 \text{ Nautical miles}$$

$$SIN\Psi = \cos \emptyset_T \cdot \sin (\lambda' - \lambda_T)/Y1$$

$$COS\Psi = -(\cos \emptyset_T \cdot \sin \emptyset' \cdot \cos (\lambda' - \lambda_T) - \sin \emptyset_T \cdot \cos \emptyset')/Y1$$

SUMMATION

This routine is entered after the measured range, Rt, the computed range, Rc, and the sine and cosine of the bearing angle, SINΨ and COSΨ, have been computed. Direction cosines and measurement factors are computed and added to the running sums.

1. The measurement is qs = Rt - Rc

   If CT = 0, 1 or 2, or if CT $\leq$ -CTU, skip to item 2.

   Otherwise test magnitude of qs.

   If state is SP, and if $|qs| \leq$ QMAX, skip to item 2.

   If state is not SP, and if $|qs| \leq$ QBAD, skip to item 2.

   Otherwise, do not enter this datum into the sample. Furthermore,

   Set NSP = NSP - 1
   Set $state_j$ to UU in the data source library

   Return to main program.

2. Set basic weight, according to source and type.

   $WT_i = 1$ master, reference or Loran data

   $WT_i = 2$ ground

   $WT_i = 4$ airborne

   Then $\zeta = WT_i$

   If source is not master or Loran, and if normal mode, or if synchronized ranging reply, add to the basic weight.

   $\zeta = \zeta + stt_j - 2stx_j \cdot SIN\Psi + 2sty_j \cdot COS\Psi$

   $w = 1/\zeta$

   If synchronized ranging reply add to sums

   $Nt = Nt + w$
   $dt = dt + w \cdot dt$
   $\zeta = WT_i$ (reset)

If source is ground or airborne, and if normal mode or if ranging reply with known position, add

$$\zeta = \zeta + sxx_j \cdot SIN\Psi^2 - 2sxy_j \cdot SIN\Psi \cdot COS\Psi + syy_j \cdot COS\Psi^2$$

3. Invert to form weighting factor

$$w = 1/\zeta$$

4. If normal mode, or if ranging reply with known position,

$$\begin{aligned}
NS &= NS + 1 \\
N &= N + w \\
cx &= cx - w \cdot SIN\Psi \\
cy &= cy + w \cdot COS\Psi \\
cxy &= cxy - w \cdot SIN\Psi \cdot COS\Psi \\
CX2 &= CX2 + w \cdot SIN\Psi \cdot SIN\Psi \\
CY2 &= CY2 + w \cdot COS\Psi \cdot COS\Psi \\
d &= d + w \cdot qs \\
dx &= dx - w \cdot qs \cdot SIN\Psi \\
dy &= dy + w \cdot qs \cdot COS\Psi
\end{aligned}$$

POSITION ADJUSTMENT 2 x 2

This subroutine provides adjustment factors DX and DY for AIRBORNE units which are in state SU or SP or when operating in the ranging mode. It is also used for feedback determination at the MASTER unit. It provides a least squares linear unbiased estimate of position error, assuming synchronized timing, by solving

$$\begin{bmatrix} DX \\ DY \end{bmatrix} = \begin{bmatrix} CX2 & cxy \\ cxy & CY2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} dx \\ dy \end{bmatrix}$$

where the terms on the righthand side of the equation are the sums of products of direction cosines and error measurements provided by the summation subroutine, which operates during the data gathering process.

1. $E1 = CX2 \cdot CY2 - (cxy)^2$

2. If $|E1| < DM2$, set DX, DY and PN to zero and exit

3. If $|E1| \geq DM2$, set PN = 1, and compute

$$DX = (dx \cdot CY2 - dy \cdot cxy)/E1$$

$$DY = (dy \cdot CX2 - dx \cdot cxy)/E1$$

POSITION AND TIME ADJUSTMENT 3 x 3

This subroutine provides the position adjustment factors DX and DY for GROUND units, the time and position adjustment factors DR, DX and DY for AIRBORNE units with state UU or UP. It continues to provide the time adjustment factor DR for AIRBORNE units in state SU or SP. It provides a least squares linear unbiased estimate of time and position errors by solving

$$\begin{bmatrix} DR \\ DX \\ DY \end{bmatrix} = \begin{bmatrix} N & cx & cy \\ cx & CX2 & cxy \\ cy & cxy & CY2 \end{bmatrix}^{-1} \begin{bmatrix} d \\ dx \\ dy \end{bmatrix}$$

where the terms on the righthand side of the equation are sums of products of direction cosines and error measurements provided by the summation subroutine, which operates during the data gathering process.

1. $E1 = CX2 \cdot CY2 - (cxy)^2$

   $E2 = cy \cdot cxy - cx \cdot CY2$

   $E3 = cx \cdot cxy - cy \cdot CX2$

   $DEL = N \cdot E1 + cx \cdot E2 + cy \cdot E3$

2. If $|DEL| < DM3$, set

   DR, DX, DY, TN, PN and GDOP to zero and exit.

3. If $|DEL| \geq DM3$, compute

$$DR = Clinv(E1 \cdot d + E2 \cdot dx + E3 \cdot dy)/DEL$$

$$GDOP = \sqrt{E1/DEL}$$

4. $X2 = N \cdot CY2 - (cy)^2$

$X3 = cx \cdot cy - N\ cxy$

$Y3 = N \cdot CX2 - (cx)^2$

$DX = (E2 \cdot d + X2 \cdot dx + X3 \cdot dy)/DEL$

$DY = (E3 \cdot d + X3 \cdot dx + Y3 \cdot dy)/DEL$

START

The sums and items that must be saved during the start-up phase are labelled and identified:

SN - Σw_i

STA - $\Sigma w_i \cdot \tau_i$

STQ - $\Sigma w_i \cdot \tau_i^2$

SDS - $\Sigma w_i \cdot Ds$

STDS - $\Sigma w_i \cdot \tau_i \cdot Ds_i$

SM - frame count after first estimate, Ds_1, obtained

TI - τ_i -- time interval

M - control parameter

τ - frame duration

Initially, items SN, STA, STQ, SDS, STDS, SM and TI are set to zero. Since the same logic that starts up system time, also starts up airborne position in x and y, we really have three sets of the above seven items. When referred to they will be subscripted -- 1 for system time, 2 for x position, and 3 for y position.

Subroutine Start is entered with Index i set to 1, 2, or 3, for t, x, or y, and has the appearance

Start

$TI_i = TI_i + \tau$

$\Delta s, \Delta\dot{s} = 0.$

If $\sigma SQ_i = 0$, there was no solution,

If $SN_i \neq 0$, then set $SM_i = SM_i + 1$.

Return.

If $SQ_i \neq 0$, then set

$w_i = 1/\sigma SQ_i$

$SN_i = SN_i + w_i$

$SDS_i = SDS_i + w_i \cdot Ds.$

If airborne unit, or if ground or reference unit with Index i = 1, then set

$STA_i = STA_i + w_i \cdot TI_i$

$STQ_i = STQ_i + w_i \cdot (TI_i)^2$

$STDS_i = STDS_i + w_i \cdot TI_i \cdot Ds$

If ground unit <u>and</u> Index i = 2 or 3, set

$STA_i, STDS_i = 0$

$STQ_i = 1.$

$SM_i = SM_i + 1.$

If $SM_i \geq M$, then compute,

$\delta s = SN_i \cdot STQ_i - (STA_i)^2$: If $\delta < 10^{-4}$, return.

$a_i = STQ_i/\delta s$

$b_i = STA_i/\delta s$

$c_i = SN_i/\delta s$

$\Delta\dot{s} = c_i \cdot STDS_i - b_i \cdot SDS_i$

$\Delta s = a_i \cdot SDS_i - b_i \cdot STDS_i + TI_i \cdot \Delta\dot{s}$

If i = 1, advance time to phase 3, and set time state to S

If i = 2, advance position to phase 3, and set position state to P.

Return.

UPDATE

This subroutine updates the covariance elements and computes effective smoothing constants. It is entered every frame while in the recursive phase, whether or not a solution is obtained. It is used by airborne units for both time and position variables, but ground and reference units use it only for variables of time. Subroutine Update is entered with Index i set to 1, 2, or 3. The indexed smoothing constants k, kp and kv have been identified in item 10 of the initialization routine on page 36. Exactly the same indexing scheme is extended to the solution variance, the effective smoothing constants and the covariance matrix as shown in Table I, following.

TABLE I

INDEXING INTERPRETATIONS

ITEM	INTERPRETATION		
	1-system time	2-x position	3-y position
a	at	ax	ay
b	bt	bx	by
c	ct	cx	cy
σSQ	σSQ_t	σSQ_x	σSQ_y
α	αt	αx	αy
β[1]	βt	βx	βy

[1]Hoping not to cause confusion, β from here on usually means (β/α).

The subroutine then proceeds.

Update

$b' = c_i \cdot \tau f + b_i$

$a' = (b' + b_i) \cdot \tau f + a_i$

If $\sigma SQ_i = 0$, then

$a_i = k_i \cdot a'$

$b_i = k_i \cdot b'$

$c_i = k_i \cdot c_i$

$\alpha_i = kp_i$

$\beta_i/\tau = kv_i/\tau$

End.

If $\sigma SQ_i \neq 0$, then

$c' = c_i + k_i(a_i \cdot c_i - b_i \cdot b_i)/\sigma SQ_i$

$\delta = 1 + k_i \cdot a'/\sigma SQ_i$

$k' = k_i/\delta$

$a_i = k' \cdot a'$

$b_i = k' \cdot b'$

$c_i = k' \cdot c'$

$\alpha_i = a_i/\sigma SQ_i$

$\beta_i/\tau = b_i/a_i$

Care must be exercised to prevent descending values of c_i from dropping below the current non-zero steady state level, which is $\sigma SQ \cdot (k_i - 1)^3/k_i^2$

End.

Return.

UPDATE GROUND

This subroutine updates the covariance elements and computes the effective smoothing constants required by a ground unit in position determination. It is entered with Index i set to 2 or 3 and uses the same indexing structure as previously described for subroutine Update in Table I. It proceeds:

Update Ground

If $\sigma SQ_i = 0$, then

$a_i = k_i \cdot a_i$

$\alpha_i = kp_i$

End.

If $\sigma SQ_i \neq 0$

$a_i = \sigma SQ_i \cdot k_i \cdot a_i / (\sigma SQ_i + k_i \cdot a_i)$

$\alpha_i = a_i / \sigma SQ_i$

End.

Return.

MODE CHECK

The mode check subroutine determines the preferred mode, based on available data sources, then considers the selected mode (switch setting) and sets the operating mode.

1. MASTER: set operating mode to NORMAL and exit.

2. REFERENCE:

If NSP $\neq$ 0,

Preferred mode is Normal
Go to 4.

If NSP = 0, check master in sight (last frame)

If MIS = 1

set MIS = 0 (off)

preferred mode is Normal, go to 4.

If MIS = 0, check for SU source

If NSU $\neq$ 0, preferred mode is RANGING, go to 4.

If NSU = 0, check own state.

If state = SP, preferred mode is NORMAL, go to 4.

If state = UP, preferred mode is the operating mode, go to 4.

3. GROUND or AIRBORNE:

If TN = 1, check sample sizes:

If NSP $\geq$ 4, preferred mode is NORMAL

If NSP = 3, then

If position state P, preferred mode is NORMAL

If position state U, preferred mode is RANGING

If TN = 0, or if NSP < 3, test for possible use of ranging mode:

If GROUND unit with position state P, preferred mode is NORMAL.

If NSP + NSU $\neq$ 0, preferred mode is RANGING.

If NSP + NSU = 0, test own state

If state synched (SU or SP), and

AIRBORNE: test NUP,

If NUP $\neq$ 0, preferred mode RANGING

If NUP = 0, preferred mode NORMAL

GROUND: test NUP,

If NUP $\geq$ 2, preferred mode RANGING

If NUP < 2, preferred mode NORMAL

If state unsynched (UP or UU), the preferred mode is the operating mode.

4. All non-master units: set operating mode.

   If preferred mode is QUIET, set operating mode to QUIET.

   (Optional - if selected mode NORMAL or RANGING, display operating mode = QUIET.)

   If mode unselected (switch setting), set operating mode to preferred mode.

   (Optional - display operating mode.)

   If preferred not QUIET, and mode not unselected, set operating mode to the selected mode.

   (Optional - If operating mode not the same as the preferred mode, display preferred mode.)

5. Return to end of cycle processing.

RANGING INTERROGATION SET-UP

A maximum of three data sources may be interrogated. Four time slots are available -- one interrogation slot and three reply slots.

The slot numbers will be paired with ID numbers in the interrogation table: SID_1, SID_2, and SID_3. Entries may have been made to this table during message processing. In any case, attempt to fill the table as much as possible, up to three entries.

1. Set NI = NSP

   If NI $\geq$ 3, set NI = 3 and skip to 3.

   If REFERENCE, and NI $\neq$ 0, skip to 3.

If NSU $\neq$ 0, add synchronized source to table.

Set NI = NI + 1

Scan data source library for source with state SU and enter IDj into interrogation table location SID_i (indexed by NI).

REFERENCE: skip to 3.

2. If NI < 3: attempt to add to table

   If NUP > 0, add source with known location.

   Set NI = NI + 1

   Scan data source library for source with state UP, and enter IDj into interrogation table location SID_i (indexed by NI)

   Set NUP = NUP - 1

   Return to 2.

   If NUP = 0, skip to 3.

   If NI $\geq$ 3, proceed.

3. If NI $\neq$ 0, make up the interrogation message and determine transmission time.

   A. Compute the transmission time, ti, radio clock.

$$ti = t \underset{\text{modulo } \tau}{+} T1(Si \overset{\text{modulo } \#}{-} S)$$

   B. Set the transmit interrupt to

$$Tir = Tip + T1(Si \overset{\text{modulo } \#}{-} S)$$

The interrogation message will contain, in the preamble, sender ID, slot Si and message type; then in the body of the message, the addressees, and reply slots, SID_1, Sr_1, SID_2, Sr_2, SID_3, and Sr_3. The interrogation table will be the form of:

SID_1	$state_1$	tj_1	Sj_1	Si	Sr_1	ti
SID_2	$state_2$	tj_2	Sj_2	Si	Sr_2	ti
SID_3	$state_3$	tj_3	Sj_3	Si	Sr_3	ti

SECTION VI

ALPHABETIC PARAMETER LISTING

PARAMETERS

The following alphabetic parameter listing is provided as a programmer's aid. An attempt is made to identify important parameters and indicate in which routine or subroutine they are computed, set or reset.

The following abbreviations are employed to indicate which routine computes, sets, alters or resets the parameter.

Abbreviation	Routine or Subroutine
D	Enter Data Phase
E	End of Frame Processing
G	Geometry
I	Initialization
I'	Reinitialization
M	Message Processing
M'	Message Preprocessing
MD	Mode Check
P	Parameters
R	Ranging Interrogation Set-Up
S	Summation
ST	Start
U	Update
UG	Update Ground
2	Position Adjustment 2 x 2
3	Position Adjustment 3 x 3
Blank	Program Constant

Following this list, certain parameters have been singled out and collected into more logical groupings for further definition and clarification.

Parameter	Routine	Description
a	I	ellipsoid, semi-major axis, converted units
a_i	ST,U,UG	variance (t, x or y)
A1	I	1/a
A2	I	a(1 - e)
A3	I	$1/(a\sqrt{1 - e})$
A4	P	$1 - e \sin^2 \phi$
A5	P	$(1 - e \sin^2 \phi)^{1/2}$
A6	P	$(1 - e \sin^2 \phi)^{3/2}$
b_i	ST,U	covariance ($t\dot{t}$, $x\dot{x}$ or $y\dot{y}$)
c		velocity of light in vacuum
c_i	ST,U	variance ($\dot{t}$, $\dot{x}$ or $\dot{y}$)
COSΨ	G	cosine, bearing angle to source
CP	I,I',E	counter for position state
CPK		cycles to advance position state
CPU		cycles to drop position state
CT	I,I',E	counter for time state
CTQ		cycles to force mode to QUIET
CTS	I,I'	cycles to advance time state
CTSG		airborne and ground CTS
CTSR		reference and master CTS

Parameter	Routine	Description
CTU	I,I'	cycles to drop time state
CTUG		airborne and ground CTU
CTUR		reference and master CTU
cx	I,I',E,S	sum of cos x, direction cosine factor
cxy	I,I',E,S	sum of (cos x)·(cos y), direction cosine factor
CX2	I,I',E,S	sum of $\cos^2 x$, direction cosine factor
cy	I,I',E,S	sum of cos y, direction cosine factor
CY2	I,I',E,S	sum of $\cos^2 y$, direction cosine factor
C1	I	mean propagation velocity
C1inv	I	1/C1
d	I,I',E,S	sum of range discrepancy measurements
DEL	3	determinant of 3 x 3 matrix
DMAX		maximum clock rate adjustment
DM2		minimum denominator size, 2 x 2 matrix
DM3		minimum denominator size, 3 x 3 matrix
DM4	I,I'	maximum DR, synchronized
DM5		trouble indication
DR	E,3	estimated timing error
Ds	E	error estimate

Parameter	Routine	Description
dt	I,I',M,E	sum of time discrepancy measurements
DTERM	I,G	D term in geodetic equation
dx	I,I',E,S	sum of x component, range discrepancy
DX	2,3	estimated west position error
dy	I,I',E,S	sum of y component, range discrepancy
DY	2,3	estimated north position error
e	I	eccentricity squared (of ellipsoid)
E1	2,3	denominator of 2 x 2 matrix, equal to cofactor used in computing DEL
E2	3	cofactor used in computing DEL
E3	3	cofactor used in computing DEL
f	I	flattening (of the ellipsoid at the poles)
FLAT		1/f
GDOP	3	geometric dilution of precision
H	I,E	current indicated heading in use
Ha	E	assumed heading
He	E	extrapolation heading, end of frame
Hi		indicated heading, wired in (optional)
Hj		source heading

Parameter	Routine	Description
HMAX		maximum heading adjustment
Ho	D	initial heading at program start
Hw	D	input wind direction
i	E	t, x or y
ID	D	unit identification
IDj	M	data source identification
IDr	M	data source identification ranging
IND	I,I',M,E	data received indicator
INDEX		index of refraction, mean atmosphere
kda		steady state β_t/α_t airborne
kdg		" " " ground
kdr		" " " reference
kdl		" " " master
kpa		position discount factor, air
kpal		steady state α_p airborne
kpg		position discount factor, ground
kpgl		steady state α_p ground
kp_1	I,I'	steady state α_t
kp_2	I,I'	" " α_x
kp_3	I,I'	" " α_y
kpm		master, position feedback smoothing

Parameter	Routine	Description
kta		time discount factor, air
kta1		steady state α_t airborne
ktg		time discount factor, ground
ktg1		steady state α_t ground
ktm		master, time feedback smoothing
ktr		time discount factor, reference
ktr1		steady state α_t reference
kt1		time discount factor, master
kt11		steady state α_t master
kva		steady state β_p/α_p airborne
kv_1	I,I'	steady state β_t/α_t
kv_2	I,I'	" " β_x/α_x
kv_3	I,I'	" " β_y/α_y
k_1	I,I'	time discount factor
k_2	I,I'	x position discount factor
k_3	I,I'	y position discount factor
M	ST	frame span in start phase
MIS	M,E,MD	master in sight indicator
MQ1		lower action level, feedback control
MQ2		upper action level, feedback control
MRE	E	mean range error

Parameter	Routine	Description
N	I,I',M,E,S	data sample size, weighted
Nc	I	mean radius of curvature at sector center
NI	R	number of ranging interrogations
NS		data sample size, unweighted
NSP	I,I',M,E	count of sources, state SP
NSU	I,I',M,E	count of sources, state SU
NT	I,I',M,E	weighted count of synched ranging replies
NUP	I,I',M,E,R	count of sources, state UP
Operating Mode	I,I',E,MD	
P		reported position Ø, λ, Z
P'		updated position Ø', λ', Z'
Pj	M	source position Øj, λj, Zj
PN	E	position data processed indicator
pos factor	E	accounts for time dependency of position solution, airborne
pos factor x	E	
pos factor y	E	
position phase	ST,E	
Preferred mode	MD	
Pt	E	transmitted position Øt, λt, Z
QMAX	I,I'	bad data limit (state SP)
QBAD		bad data limit (state not SP)
qs	S	measurement of range discrepancy

Parameter	Routine	Description
Rc	G	computed range or position difference
Rcinv	G	1/Rc
Rcj	M	computed range in data source library
$\overline{\text{rinv}}$	P	inverse mean radius of curvature
RIS	I,I',M	Reference in Sight
Rm	P	meridional radius of curvature
Rs	G	surface range
RS2	G	$(Rs)^2$
Rt	M	timed, or measured, one way range
S	I,I'	position reporting slot
Sa	D,M	slot assignment
SDS_i	ST	$\Sigma w_i \cdot Ds$
Selected Mode	D	
Si	I,I',R	ranging interrogation slot (1,2,3)
SID	M,R	ID of potential interrogee (1,2,3)
SIGN	E	sign agreement (DR and MRE)
SINΨ	G	sine of bearing angle to source
Sj	M,R	source slot number
SM_i	E,ST	frame count since 1st estimate
SMA		ellipsoid, semi-major axis
SN_i	ST	Σw_i

Parameter	Routine	Description
Sr	I,I',MR,	ranging reply slot (1,2,3)
state	I,I',E	unit time and position status
state_j	M,R	source state
$STDS_i$	ST	$\Sigma w_i \cdot \tau_i \cdot Ds_i$
STQ_i	ST	$\Sigma w_i \cdot \tau_i^2$
STT	E	time variance
STX	E	time-longitude covariance
STY	E	time-latitude covariance
SXX	E	longitude variance
SXY	E	longitude-latitude covariance
SYY	E	latitude variance
t	I,I',E	position reporting time, radio clock
$\dot{t}$	I,E	assumed clock rate
T'	M	adjusted time measurement
TAN	P	$(\tan \emptyset) \sqrt{1 - e \sin^2 \emptyset}/2a$
Task	D	unit task designation
Task_j	D	source task designation
tc	I	current computer clock reading
tcomp	M',M	computer clock, first message in cycle
Teoc	I,I',E	interrupt time for end of frame processing
TF	I,I',M,E	feedback time adjustment

Parameter	Routine	Description
ti	R	ranging interrogation time, radio clock
TI_i	ST,E	time interval since 1st estimate
Tip	E	interrupt time, position tx, computer clock
Tir	E	interrupt time, ranging interr., computer clock
tj	M,R	message received time, radio clock
TN	E	timing data processed indicator
to	I	computer clock reading at program start
tq	M	mean time, ranging interrogation and reply
tr	M	ranging reply received time
trad	M'	radio clock reading, first message in cycle
time factor x	E	accounts for position dependency of time solution, ground
time factor xy	E	accounts for position dependency of time solution, ground
time factor y	E	accounts for position dependency of time solution, ground
time phase	E,ST	
Trr	M	interrupt time, other tx, computer clock
Ts	M	measured transmit time interval
Tsr	M	transmit time reported in ranging reply
txp	I,I',M	extrapolation time

Parameter	Routine	Description
T1	I,E	assumed slot duration, local radio clock
T2	I,E	assumed frame duration, local radio clock
V	I,E	current indicated speed in use
Va	E	assumed speed
Ve	E	extrapolation speed, end of frame
Vi		indicated speed, wired in (optional)
Vj		source speed
VMAX		maximum speed adjustment
Vo	D	initial velocity at program start
Vx	I,E	speed west
Vy	I,E	speed north
Vw	D	input wind speed
$V\lambda$	I,E	longitudinal speed west
VØ	I,E	latitudinal speed north
w_i	ST	weighting factor
WT_i	S	weighting factor
Wx	I,E	wind speed west
Wy	I,E	wind speed north
x	G	spheroidal component of Rs, east
XF	I,I',M,E	position, west, feedback adjustment
X2	3	cofactor used in computing DEL
X3	3	cofactor used in computing DEL

Parameter	Routine	Description
y	G	spheroidal component of Rs, north
YF	I,I',M,E	position, east, feedback adjustment
Y3	3	cofactor used in computing DEL
Z	I,E	reported altitude
Za		mean airborne altitude
Zc		altitude of sector center
Zi		indicated altitude
Zj	M	source altitude
Zo	D,I	initial altitude
α_i	E,U,UG	position on time smoothing constant
β_i	ST,U	rate smoothing constant
δ		internal delay
δB		transmission interrupt safety margin
ΔC	I,I',M,E	change in clock advance
Δd	E	time rate estimate adjustment
δp		end of frame processing delay
δq	E	indicated velocity change
ΔS	I,I'	slot adjustment
Δs	E,ST	time or position adjustment
$\Delta\dot{s}$	E,ST	rate adjustment
Δt	E	timing estimate adjustment
ΔX	E	position estimate adjustment, west

Parameter	Routine	Description
ΔY	E	position estimate adjustment, north
$\Delta\lambda$	E	longitude adjustment
$\Delta\phi$	E	latitude adjustment
λ	I,I',E	current longitude
λ'	I,M,E	updated longitude
λc		longitude of sector center (relative)
λj	M	longitude of source (relative)
λo	D,I	initial longitude (relative)
λr		reference longitude
λt	I,M,E	transmitted longitude (relative)
λ_T	G	longitude of Loran transmitter
νp	VP	propagation velocity estimate
σSQ_i	E,2,3	solution variance
τ		frame duration
τa	E	extrapolation time
ϕ	I,I',E	current latitude
ϕ'	I,M,E	updated latitude
ϕc		latitude of sector center (relative)
ϕj	M	latitude of data source (relative)
ϕo	D,I	initial latitude (relative)
ϕr		reference latitude

Parameter	Routine	Description
$\emptyset t$	I,M,E	transmitted latitude (relative)
$\emptyset_T$	G	latitude of Loran transmitter
#		number of slots per frame

PROGRAM CONSTANTS

Constant	Nominal Values	Description
c	299776 kilometers/sec 161866.091 n.m./sec	velocity of light in vacuum
CPK	64	consecutive frames with data, advance position state to P, air and ground
CPU	30	consecutive frames no data, drop position state to U, air only
CTQ	3600	consecutive frames no data, force mode to QUIET
CTSG CTSR	24 (air,ground) 60 (ref,master) }	consecutive frames with data, advance time state to S
CTUG CTUR	56 (air,ground) 60 (ref,master) }	consecutive frames no data, drop time state to U
DMAX	10 nanosec/sec	maximum clock rate adjustment, synchronized
DM2	10^{-4}	minimum denominator size, 2 x 2 position adjustment
DM3	10^{-6}	minimum denominator size, 3 x 3 time and position adjustment
DM4	{ 3 μs (air,master) 1 μs (ground,ref)	max DR, synchronized
DM5	1 μs	trouble indication

Constant	Nominal Values	Description
FLAT	298.25	1/f defines flattening
HMAX	20 degrees	maximum heading adjustment, before velocity smoothing
INDEX	1.00029	index of refraction, mean atmosphere
QBAD	500 n.m.	maximum discrepancy for non SP units
QMAX	{ 3 nm (air,master) 1/4 nm (ground,ref)	max discrepancy for SP units
SMA	6,378,165 meters	ellipsoid, semi-major axis
VMAX	10 knots	maximum speed adjustment, before velocity smoothing
Za	10,000 feet	mean airborne altitude
Zc	200 feet	altitude of sector center above sea level
δ	24 μsec	internal delay (site sensitive)
δB	0.002 seconds	transmission interrupt safety margin
δp	0.06 seconds	end of frame processing delay
λc	5 minutes	sector center relative longitude, with respect to reference
λr	71° 15' west +4275 minutes	reference longitude
MQ1	0	lower action level, feedback control
MQ2	20 nanosec	upper action level, feedback control

Constant	Nominal Values	Description
τ	1 second	frame duration
Øc	3 minutes	sector center relative latitude with respect to reference
Ør	42° 30' North (+2550 minutes)	reference latitude
#	100	number of slots per frame

SMOOTHING CONSTANTS

Constant	Nominal Values	Description
kda	.0006	steady state β_t/α_t airborne
kdg	.0027	steady state β_t/α_t ground
kdr	.0083	steady state β_t/α_t reference
kdl	.0083	steady state β_t/α_t master
kpa	1.15	position discount factor, airborne
kpal	.244	steady state α_p airborne
kpg	1.02	position discount factor, ground
kpgl	.0196	steady state α_p ground
kpm	.3	master, position feedback
kta	1.025	time discount factor, airborne
ktal	.0482	steady state α_t airborne
ktg	1.05	time discount factor, ground
ktgl	.093	steady state α_t ground
ktm	.3	master, time feedback

Constant	Nominal Values	Description
ktr	1.1	time discount factor, reference
ktrl	.17	steady state α_t reference
ktl	1.1	time discount factor, master
ktll	.17	steady state α_t master
kva	.017	steady state α_p/β_p airborne

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